1. Let $\mathbf{u_1} = (3, -1, 2)$ and $\mathbf{u_2} = (3, 1, 5)$.
   (a) Express the vector $\mathbf{v} = (9, 11, 27)$ as a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$ if possible.
   (b) Find k such that the vector $\mathbf{w} = (-5, 4, k)$ is a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$.

2. Let $\mathbf{a}_1 = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 9 \\ -3 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 7 \\ 6 \\ h \end{bmatrix}$.
   (a) Find h so that $\mathbf{b}$ is in $\mathrm{Span}\{\mathbf{a}_1, \mathbf{a}_2\}$.
   (b) For the h that you found in the previous part, express $\mathbf{b}$ as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$.

3. Let $\mathbf{b}_1 = (h, 5, 7)$, $\mathbf{b}_2 = (-1, 3, 7)$, and $\mathbf{b}_3 = (1, 1, 2)$.
   Find h so that $\mathbf{b}_3 \in \mathrm{Span}\{\mathbf{b}_1, \mathbf{b}_2\}$.

4. (a) Express the plane $x - 3y + 4z = 0$ as a span of vectors.
   (b) Express the intersection of the two planes $x - 3y + 4z = 0$ and $2x + z = 0$ as a span of vectors.

5. Find an equation of the plane in form $Ax + By + Cz = D$ that is spanned by the vectors $(2, 3, -1)$ and $(4, 1, 5)$.

6. Let $\mathbf{u_1} = (2, 0, 3, -1)$, $\mathbf{u_2} = (-4, 0, -6, 2)$, $\mathbf{u_3} = (5, 5, 0, 3)$, $\mathbf{u_4} = (1, 3, -6, 5)$, $\mathbf{0} = (0, 0, 0, 0)$.
   Determine whether each set is linearly independent or linearly dependent.
   (a) $\{\mathbf{u_1}\}$
   (b) $\{\mathbf{u_1}, \mathbf{u_2}\}$
   (c) $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$
   (d) $\{\mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$
   (e) $\{\mathbf{u_3}, \mathbf{u_4}\}$
   (f) $\{\mathbf{u_3}, \mathbf{u_4}, \mathbf{0}\}$

7. Let $\mathbf{u_1} = (5, 2, -1, 6)$, $\mathbf{u_2} = (3, 1, 0, 2)$,
   $\mathbf{u_3} = (1, 1, -2, 6)$, $\mathbf{u_4} = (1, 1, -2, 1)$, $\mathbf{u_5} = (1, 0, 0, 0)$.
   Determine whether each set is linearly independent or linearly dependent.
   (a) $\{\mathbf{u_1}, \mathbf{u_2}\}$
   (b) $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$
   (c) $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}\}$
   (d) $\{\mathbf{u_2}, \mathbf{u_3}, \mathbf{u_4}, \mathbf{u_5}\}$
   (e) $\{\mathbf{u_3}, \mathbf{u_4}, \mathbf{u_5}\}$
   (f) $\{\mathbf{u_5}\}$

8. Let $\mathbf{u_1} = (2, 3, -1,)$, $\mathbf{u_2} = (5, 4, -1)$, $\mathbf{u_3} = (5, -3, 2)$, $\mathbf{u_4} = (0, 6, -2)$, $\mathbf{u_5} = (0, -15, 5)$.
   Determine whether each set is linearly independent or linearly dependent. In each case, state whether the span of the set is a point, line, plane, or $\mathbb{R}^3$.
   (a) $\{\mathbf{u}_1, \mathbf{u}_2\}$
   (b) $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$
   (c) $\{\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$
   (d) $\{\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$
   (e) $\{\mathbf{u}_4, \mathbf{u}_5\}$

9. Let $\mathbf{u_1} = (0, -5, 5,)$, $\mathbf{u_2} = (0, 3, -3)$, $\mathbf{u_3} = (1, 1, 1)$, $\mathbf{u_4} = (1, 0, 1)$, $\mathbf{u_5} = (2, 2, 0)$, and $\mathbf{0} = (0, 0, 0)$.
   Determine whether each set is linearly independent or linearly dependent. (LI or LD?) In each case, state whether the span of the set is a point, line, plane, or $\mathbb{R}^3$.
   (a) $\{\mathbf{u}_1\}$
   (b) $\{\mathbf{u}_1, \mathbf{u}_2\}$
   (c) $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$
   (d) $\{\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$
   (e) $\{\mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4, \mathbf{u}_5\}$
   (f) $\{\mathbf{u}_4, \mathbf{u}_5\}$
   (g) $\{\mathbf{u}_4, \mathbf{u}_5, \mathbf{0}\}$
   (h) $\{\mathbf{0}\}$

10. Determine if the following sets are subspaces. For those that are, express the set as a span of vectors. For those that are not, provide a counter-example to show it is not closed under VA or SM.
   (a) $S = \{(x, y, z) \in \mathbb{R}^3 |\ x = 4s - t,\ y = s + 3t, z = 6s\}$
   (b) $S = \{(x, y, z) \in \mathbb{R}^3 |\ 3x + 4y - z = 2\}$
   (c) $S = \{(x, y, z) \in \mathbb{R}^3 |\ z^2 = xy\}$
   (d) $S = \{(x, y, z) \in \mathbb{R}^3 |\ x + 2y - 3z = 0\}$
   (e) $S = \{(x, y, z) \in \mathbb{R}^3 |\ y \geq x\}$
   (f) $S = \{(x, y) \in \mathbb{R}^2 |\ x = 4 + 2t, y = -6 - 3t\}$
   (g) $S = \{(x, y, z) \in \mathbb{R}^3 |\ y + z \geq -1\}$
   (h) $S = \{(x, y, z) \in \mathbb{R}^3 |\ z = 2x - 3y\}$
   (i) $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \ \middle|\ xy + z = 0 \right\}$
   (j) $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \ \middle|\ 2x = y - z \right\}$
   (k) $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \ \middle|\ x = 6t,\ y = 4t \text{ some } t \in \mathbb{R} \right\}$

11. Let $\mathbf{a}_1 = \begin{bmatrix}1\\2\end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix}2\\1\end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix}3\\5\end{bmatrix}$.

(a) Express $\mathbf{a}_3$ as linear combinations of $\mathbf{a}_1$ and $\mathbf{a}_2$ if possible.

(b) Is $\{\mathbf{a}_1\}$ a basis for $\mathbb{R}^2$? Why or why not?

(c) Is $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ a basis for $\mathbb{R}^2$? Why or why not?

(d) Is $\{\mathbf{a}_2, \mathbf{a}_3\}$ a basis for $\mathbb{R}^2$? Why or why not?

12. Let $\mathbf{u_1} = (4, 2, 5)$, $\mathbf{u_2} = (3, -1, -2)$, and $\mathbf{u_3} = (6, 2, 0)$

(a) Is $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ a basis for $\mathbb{R}^3$? Justify.

(b) Is it possible to express $\mathbf{u_3}$ as a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$? Justify without solving.

(c) Is it possible to express the vector $(9, 5, 2)$ as a linear combination of $\mathbf{u_1}$, $\mathbf{u_2}$, and $\mathbf{u_3}$? Justify without solving.

13. Let $A = \begin{bmatrix}2 & 1 & 0\\3 & 1 & 1\\1 & 0 & 1\end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$.

(a) Is $\mathbf{v}$ in $\mathrm{Nul}(A)$? Justify your answer.

(b) Is $\mathbf{v}$ in $\mathrm{Col}(A)$? Justify your answer.

14. Let $A = \begin{bmatrix}1 & -1 & 0\\1 & -1 & 0\\0 & 0 & 0\end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix}2\\2\\0\end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix}0\\0\\2\end{bmatrix}$.

(a) Find a basis for $\mathrm{Col}(A)$.

(b) Find a basis for $\mathrm{Nul}(A)$.

(c) Is $\mathbf{v}_1$ in $\mathrm{Nul}(A)$? Justify your answer.

(d) Is $\mathbf{v}_1$ in $\mathrm{Col}(A)$? Justify your answer.

(e) Is $\mathbf{v}_2$ in $\mathrm{Nul}(A)$? Justify your answer.

(f) Is $\mathbf{v}_2$ in $\mathrm{Col}(A)$? Justify your answer.

15. Let $\mathbf{a_1} = (2, 3, -1, 1)$, $\mathbf{a_2} = (-2, -3, 1, -1)$,
$\mathbf{a_3} = (2, 3, 1, 5)$, $\mathbf{a_4} = (2, 3, 2, 7)$, $\mathbf{a_5} = (4, 6, 3, 12)$.
Find a basis for $S = \mathrm{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$.

16. The matrix $A = \begin{bmatrix}2 & -6 & 5 & 3 & -8 & 18\\-3 & 9 & -1 & -5 & -1 & -36\\0 & 0 & 4 & 8 & -8 & 36\end{bmatrix}$

has reduced form $R = \begin{bmatrix}1 & -3 & 0 & 0 & 1 & 4\\0 & 0 & 1 & 0 & -2 & -1\\0 & 0 & 0 & 1 & 0 & 5\end{bmatrix}$.

(a) Choose a basis for $\mathrm{Col}(A)$ from the columns of A.

(b) Choose a basis for $\mathrm{Nul}(A)$.

17. The matrix $A = \begin{bmatrix}5 & 4 & 1 & 0 & 1 & 13\\4 & 5 & -1 & 0 & 1 & 14\\-4 & -4 & 0 & 0 & 0 & -12\\3 & 2 & 1 & 0 & 1 & 7\end{bmatrix}$

reduces to $R = \begin{bmatrix}1 & 0 & 1 & 0 & 0 & 1\\0 & 1 & -1 & 0 & 0 & 2\\0 & 0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$.

(a) Choose a basis for $\mathrm{Col}(A)$ from the columns of A.

(b) Choose a basis for $\mathrm{Nul}(A)$.

18. The matrix $A = \begin{bmatrix}6 & -9 & 2 & -12 & 1 & 8\\-6 & 9 & 5 & 54 & 2 & -1\\8 & -12 & 1 & -26 & 0 & 9\\0 & 0 & 3 & 18 & 3 & 3\end{bmatrix}$

reduces to $R = \begin{bmatrix}1 & -3/2 & 0 & -4 & 0 & 1\\0 & 0 & 1 & 6 & 0 & 1\\0 & 0 & 0 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$

Let $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}, \mathbf{a_4}, \mathbf{a_5}, \mathbf{a_6}$ be the columns of A.

(a) Choose a basis for $\mathrm{Col}(A)$ from the columns of A.

(b) Express each column of A that is not in your basis as a linear combination of your basis vectors.

(c) Find a basis for $\mathrm{Nul}(A)$.

19. The matrix $\begin{bmatrix}3 & 3 & a & c & 1 & e\\b & 2 & -8 & 6 & f & 15\\0 & d & 0 & 2 & 1 & 6\end{bmatrix}$

has reduced form $\begin{bmatrix}1 & 0 & 4 & -1 & 0 & 0\\0 & 1 & 0 & 2 & 0 & 5\\0 & 0 & 0 & 0 & 1 & 1\end{bmatrix}$.

Find a, b, c, d, e, and f.

20. The matrix $\begin{bmatrix}2 & a & 5 & 3 & b & c\\d & 9 & e & -5 & -1 & -36\\0 & 0 & 4 & f & -8 & 36\end{bmatrix}$

has reduced form $\begin{bmatrix}1 & -3 & 0 & 0 & 1 & 4\\0 & 0 & 1 & 0 & -2 & -1\\0 & 0 & 0 & 1 & 0 & 5\end{bmatrix}$.

Find a, b, c, d, e, and f.

21. Suppose A is $n \times m$, $\mathrm{Dim}(\mathrm{Col}(A)) = 2$, $\mathrm{Dim}(\mathrm{Nul}(A)) = 3$ and $\mathrm{Dim}(\mathrm{Nul}(A^T)) = 4$. Find n and m.

22. Suppose A is an 5×8 matrix.
   (a) What is the minimum nullity of A?
   (b) Can the system $A\mathbf{x} = \mathbf{0}$ have a unique solution?
   (c) What is minimum nullity of A^T?
   (d) Can the system $A^T\mathbf{x} = \mathbf{0}$ have a unique solution?

23. Suppose A is a 6×4 matrix, and that the nullity of A^T is 3.
   (a) Find the nullity of A.
   (b) Does the system $A\mathbf{x} = \mathbf{0}$ have a unique solution?
   (c) Are the columns of A linearly independent?

24. Suppose A is 5×3 and the general solution to the equation $A\mathbf{x} = \mathbf{b}$ is given by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + s \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$.
   (a) Find the general solution to $A\mathbf{x} = \mathbf{0}$.
   (b) Find the rank of A.

25. Suppose $\mathrm{Nul}(A) = \mathrm{Span}\left\{ \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \right\}$ for a matrix A, and that $\mathbf{u} = \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix}$ is one particular solution to $A\mathbf{x} = \mathbf{b}$.
   What is the general (parametric) solution to $A\mathbf{x} = \mathbf{b}$?

26. Suppose the general solution to $A\mathbf{x} = \mathbf{b}$ is given by
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}.$$
   (a) Find a non-zero solution to the homogeneous equation $A\mathbf{x} = \mathbf{0}$
   (b) Find the general solution to $A\mathbf{x} = 2\mathbf{b}$.
   (Hint: A(2**x**)=2(A**x**).)

27. Let $\mathbf{u} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 6 \\ -4 \\ 3 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.
   Suppose A is 7×3, and that $\mathrm{Nul}(A) = \mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$.
   (a) Find the nullity of A.
   (b) Find the rank of A.
   (c) Give the general solution of $A\mathbf{x} = \mathbf{0}$ in parametric form.
   (d) Give the general solution of $A\mathbf{x} = A\mathbf{w}$ in parametric form.

ANSWERS ON NEXT PAGE.

ANSWERS:

1. (a) $\mathbf{v} = -4\mathbf{u_1} + 7\mathbf{u_2}$.
   (b) $k = 1/6$.
2. (a) $h = 5$.
   (b) $\mathbf{b} = \frac{5}{3}\mathbf{a}_1 - \frac{1}{3}\mathbf{a}_2$.
3. $h = 17$.
4. (a) The plane is given by $\text{Span}\left\{\begin{bmatrix}3\\1\\0\end{bmatrix}, \begin{bmatrix}-4\\0\\1\end{bmatrix}\right\}$.
   (b) The line is given by $\text{Span}\left\{\begin{bmatrix}-1/2\\7/6\\1\end{bmatrix}\right\}$.
5. $8x - 7y - 5z = 0$ or any non-zero multiple.
6. (a) LI.
   (b) LD.
   (c) LD.
   (d) LI.
   (e) LI.
   (f) LD.
7. (a) LI.
   (b) LD.
   (c) LD.
   (d) LI.
   (e) LI.
   (f) LI.
8. (a) LI. Plane.
   (b) LD. Plane.
   (c) LI. $\mathbb{R}^3$.
   (d) LD. $\mathbb{R}^3$.
   (e) LD. Line.
9. (a) LI. Line.
   (b) LD. Line.
   (c) LD. Plane
   (d) LI. $\mathbb{R}^3$.
   (e) LD. $\mathbb{R}^3$.
   (f) LI. Plane.
   (g) LD. Plane
   (h) LD. Point.
10. (a) Yes. $S = \text{Span}\left\{\begin{bmatrix}4\\1\\6\end{bmatrix}, \begin{bmatrix}-1\\3\\0\end{bmatrix}\right\}$.
    (b) No. Not closed under VA or SM. C-E: Let $\mathbf{u} = (1, 0, 1)$. $\mathbf{u} \in S$, but $2\mathbf{u} = \mathbf{u} + \mathbf{u} \notin S$.
    (c) No. Not closed under VA. C-E: Let $\mathbf{u} = (1, 0, 0)$. Let $\mathbf{v} = (0, 1, 0)$. The vectors $\mathbf{u}$ and $\mathbf{v}$ are in S, but $\mathbf{u} + \mathbf{v} = (1, 1, 0) \notin S$.
    (d) Yes. $S = \text{Nul}\begin{bmatrix}1 & 2 & -3\end{bmatrix} = \text{Span}\left\{\begin{bmatrix}-2\\1\\0\end{bmatrix}, \begin{bmatrix}3\\0\\1\end{bmatrix}\right\}$.
    (e) No. S is not closed under SM. C-E: Let $\mathbf{u} = (0, 1, 0)$ and $k = -1$. The vector $\mathbf{u} \in S$, but $k\mathbf{u} = -1(0, 1, 0) = (0, -1, 0) \notin S$.
    (f) Yes. $S = \text{Span}\{(2, -3)\}$.
    (g) No. Not closed under SM or VA. C-E: Let $\mathbf{u} = (0, 0, -1)$ and $k = 2$. The vector $\mathbf{u} \in S$, but $k\mathbf{u} = \mathbf{u} + \mathbf{u} \notin S$.
    (h) Yes. $S = \text{Span}\left\{\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}0\\1\\-3\end{bmatrix}\right\}$.
    (i) No. Not closed under SM or VA. C-E: Let $\mathbf{u} = (1, 1, -1)$ and $k = 2$. The vector $\mathbf{u} \in S$, but $k\mathbf{u} = \mathbf{u} + \mathbf{u} = (2, 2, -2) \notin S$.
    (j) Yes. $S = \text{Span}\left\{\begin{bmatrix}1\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$.
    (k) Yes. $S = \text{Span}\left\{\begin{bmatrix}6\\4\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$.
11. (a) $\mathbf{a}_3 = \frac{7}{3}\mathbf{a}_1 + \frac{1}{3}\mathbf{a}_2$.
    (b) No, since $\text{Span}\{\mathbf{a}_1\} \neq \mathbb{R}^2$. (Takes at least two vectors...)
    (c) No, since $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ is linearly dependent. (...and no more than two.)
    (d) Yes, since $\{\mathbf{a}_2, \mathbf{a}_3\}$ is linearly independent and spans $\mathbb{R}^2$.
12. (a) Yes, since $\begin{vmatrix}4 & 3 & 6\\2 & -1 & 2\\5 & -2 & 0\end{vmatrix} \neq 0$.
    (b) No, since $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is linearly independent.
    (c) Yes, since $\text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} = \mathbb{R}^3$.
13. (a) No. $A\mathbf{v} \neq \mathbf{0}$.
    (b) Yes. $A\mathbf{x} = \mathbf{v}$ is consistent.
14. Let $A = \begin{bmatrix}1 & -1 & 0\\1 & -1 & 0\\0 & 0 & 0\end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix}2\\2\\0\end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix}0\\0\\2\end{bmatrix}$.
    (a) $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}\right\}$ (b) $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$
    (c) Yes, $A\mathbf{v}_1 = \mathbf{0}$.
    (d) Yes, $\mathbf{v}_1 = 2\mathbf{a}_1$, double the first column of A.
    (e) Yes, $A\mathbf{v}_2 = \mathbf{0}$.
    (f) No, $A\mathbf{x} = \mathbf{v}_2$ has no solution.
15. Basis for S: $\left\{\begin{bmatrix}2\\3\\-1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\1\\5\end{bmatrix}\right\}$.

16. (a) Basis for $\text{Col}(A)$: $\left\{ \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix} \right\}$

(b) Basis for $\text{Nul}(A)$: $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 1 \\ -5 \\ 0 \\ 1 \end{bmatrix} \right\}$

17. (a) Basis for $\text{Col}(A)$: $\left\{ \begin{bmatrix} 5 \\ 4 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(b) Basis for $\text{Nul}(A)$: $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

18. (a) Basis for $\text{Col}(A)$: $\{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_5\}$.

(b) $\mathbf{a}_2 = -\frac{3}{2}\mathbf{a}_1$, $\mathbf{a}_4 = -4\mathbf{a}_1 + 6\mathbf{a}_3$, $\mathbf{a}_6 = \mathbf{a}_1 + \mathbf{a}_3$.

(c) Basis for $\text{Nul}(A)$: $\left\{ \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

19. $(a, b, c, d, e, f) = (12, -2, 3, 1, 16, 5)$.

20. $(a, b, c, d, e, f) = (-6, -8, 18, -3, -1, 8)$.

21. $n = 6, m = 5$.

22. (a) Max Rank of A = 5, so Min Nullity of $A = 8 - 5 = 3$.

(b) No. Solution must have at least 3 parameters.

(c) Min Nullity of A^T is 0.

(d) Yes. Solution is unique when Nullity of A is 0.

23. (a) 1.

(b) No. There will be one parameter in the solution.

(c) No. There is a non-trivial solution to $A\mathbf{x} = \mathbf{0}$.

24. (a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$=s$\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$+t$\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

(b) Rank of $A = 1$.

25. $\left\{ \begin{array}{lllllll} x & = & 6 & - & 3s & + & 5t \\ y & = & -1 & + & 4s & & \\ z & = & 2 & + & 2s & + & t \end{array} \right.$

26. (a) $(x, y, z) = (1, 0, -2)$, for example when $s = 1$ and $t = 0$.

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$.

27. (a) 2.

(b) 1.

(c) $\left\{ \begin{array}{lll} x & = & -2s + 5t \\ y & = & s \\ z & = & t \end{array} \right.$

(d) $\left\{ \begin{array}{lll} x & = & 6 - 2s + 5t \\ y & = & -4 + s \\ z & = & 3 + t \end{array} \right.$