1. Solve the linear program by graphing the feasibility region.
$$\begin{array}{ll} \textbf{Minimize} & z = 7x - 14y \\ \text{subject to} & 3x + 4y \leq 12 \\ & x - y \geq 0 \\ & 0 \leq x, \quad 0 \leq y \end{array}$$

2. Solve the linear program by graphing the feasibility region.
$$\begin{array}{ll} \textbf{Maximize} & z = -x + y \\ \text{subject to} & x + y \geq 9 \\ & x - 4y \geq 4 \\ & 2x + y \leq 26 \\ & x \geq 0, \quad y \geq 0 \end{array}$$

3. Solve the linear program by graphing the feasibility region.
$$\begin{array}{ll} \textbf{Maximize} & z = 7x - 4y \\ \text{subject to} & 7x - 3y \geq 4 \\ & 7x + y \leq 57 \\ & y \geq 1 \\ & y \leq 8 \end{array}$$

4. Solve the linear program by graphing the feasibility region.
$$\begin{array}{ll} \textbf{Minimize} & z = 3x - 2y \\ \text{subject to} & 2x + y \geq 4 \\ & x - y \leq 4 \\ & -3x + 7y \leq 0 \\ & x \geq 0, \quad y \geq 0 \end{array}$$

5. Solve the linear program by graphing the feasibility region.
$$\begin{array}{ll} \textbf{Minimize} & z = x - y \\ \text{subject to} & x + 2y \geq 16 \\ & 2x - y \geq 12 \\ & y \leq 12 \\ & x \geq 0, \quad y \geq 0 \end{array}$$

6. Solve using the simplex algorithm.
$$\begin{array}{ll} \textbf{Maximize} & z = 3x_1 - 4x_2 + 5x_3 \\ \text{subject to} & 2x_1 - 3x_2 + 5x_3 \leq 30 \\ & -2x_1 + x_2 + x_3 \leq 20 \\ & x_1 + 7x_2 \leq 10 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

7. Solve using the simplex algorithm.
$$\begin{array}{ll} \textbf{Maximize} & z = 20x_1 + 5x_2 - 13x_3 \\ \text{subject to} & x_1 + 4x_2 - x_3 \leq 20 \\ & 2x_1 + 3x_2 - 2x_3 \leq 30 \\ & -4x_1 - 9x_2 + 18x_3 \leq 80 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

8. Solve using the simplex algorithm.
$$\begin{array}{ll} \textbf{Minimize} & z = -5x_1 + 4x_2 - 3x_3 \\ \text{subject to} & x_1 + x_2 - 2x_3 \leq 10 \\ & 7x_2 + x_3 \leq 5 \\ & 5x_1 - 3x_2 + 2x_3 \leq 15 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

9. Solve using the simplex algorithm.
$$\begin{array}{ll} \textbf{Minimize} & z = -4x_1 + 3x_2 - 2x_3 \\ \text{subject to} & 4x_1 + 5x_2 \leq 2 \\ & 8x_1 - 2x_2 + 3x_3 \leq 8 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

10. The linear program below has no maximum. Use the simplex method to find a feasible solution where $z \geq 1300$.
$$\begin{array}{ll} \textbf{Maximize} & z = -2x_1 + 4x_2 + x_3 \\ \text{subject to} & 5x_1 + x_2 - 3x_3 \leq 3 \\ & 6x_1 - 2x_2 + 4x_3 \leq 7 \\ & x_1 \geq 0, \quad x_2 \geq 0 \quad x_3 \geq 0 \end{array}$$

11. The linear program below has no minimum. Use the simplex method to find a feasible solution where $z = -2012$.
$$\begin{array}{ll} \textbf{Minimize} & z = 3x_1 - 6x_2 + 2x_3 \\ \text{subject to} & x_1 + 2x_2 - 5x_3 \leq 3 \\ & 2x_2 - x_3 \leq 2 \\ & -5x_1 - 2x_2 + 2x_3 \leq 4 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

12. The linear program below has no minimum. Use the simplex method to find a feasible solution where $z = -7006$.
$$\begin{array}{ll} \textbf{Minimize} & z = -3x_1 + 4x_2 + 2x_3 \\ \text{subject to} & -4x_1 + x_2 + x_3 \leq 3 \\ & 2x_1 - 5x_2 + x_3 \leq 4 \\ & -2x_1 \quad x_3 \leq 1 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

13. The linear program below has no maximum. Use the simplex method to find a feasible solution where $z = 5012$.
$$\begin{array}{ll} \textbf{Maximize} & z = -3x_1 + 4x_2 + 3x_3 \\ \text{subject to} & -4x_1 + 2x_2 + x_3 \leq 8 \\ & -2x_1 + x_2 - x_3 \leq 3 \\ & x_1 - 3x_2 \leq 6 \\ & x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \end{array}$$

14. Consider the linear program :
$$\begin{array}{ll} \textbf{Maximize} & z = x_1 + x_2 \\ \text{subject to} & x_1 + 3x_2 \leq 15 \\ & 2x_1 + x_2 \leq 10 \\ & x_1 \leq 4 \\ & x_1 \geq 0, \quad x_2 \geq 0 \end{array}$$

   (a) Solve using the graphical method.

   (b) Solve using the simplex algorithm.

   (c) In the previous part, label your simplex tables A, B, C etc. (A is the initial table, B is the second table, and so on.) Using the same labels, indicate the points on your graph that correspond to the basic feasible solution for each table.

15. You are selling martinis at an event to raise funds for charity. A dry martini is $5/3$ oz. of Gin and $1/3$ oz. of Vermouth. A medium martini is $3/2$ oz. Gin and $1/2$ oz. Vermouth. You have available 60 oz. of Gin and 16 oz. of Vermouth. You can sell dry martinis for \$8 each and medium martinis for \$10 each. How many of each kind of martini should be sold to maximize revenue? Name your variables, set up a linear program, and solve by sketching the feasibility region.

16. The Simple Machine Company makes Widgets, Gadgets, and Gizmos out of pulleys, wedges, and levers. Each Widget requires 3 pulleys, 2 wedges, and 4 levers. Each Gadget requires 7 pulleys and 1 lever. Each Gizmo requires 3 pulleys, 2 wedges, and 5 levers. The company has 56 pulleys, 16 wedges, and 33 levers available. Suppose the company sells Widgets for \$2 each, Gadgets for \$4 each, and Gizmos for \$1 each. Set-up and solve a linear program to determine how many Widgets, Gadgets, and Gizmos should be made to maximize revenue. What is the maximum revenue? When revenue is maximized, how many pulleys, wedges, and levers are left over?

17. The Furniture Factory makes beds, chairs, and couches from the raw materials labor, lumber, and cloth. The company makes a profit of \$60 per bed, \$10 per chair, and \$40 per couch. Each bed requires 1 hour of labor, 3 metres of lumber, and 3 metres of cloth; each chair requires 1 hour of labor, 5 meters of lumber and 7 meters of cloth; and each couch requires 2 hours of labor, 1 meter of lumber, and 1 meter of cloth. If there are 100 hours of labor, 90 meters of lumber, and 120 meters of cloth available, you would like to determine how many beds, chairs, and couches the company should make to maximize profit.
    (a) Name variables and set up a linear program that represents this situation.
    (b) What is the maximum profit?
    (c) How many beds, chairs, and couches should be made to maximize profit?
    (d) When profit is maximized, how much labor, lumber, and cloth will go unused?

18. Squeeky Cleaners makes three cleaning products: Ocean Fresh, Summer Breeze, and Lemon Zest, which sell for \$1, \$3, and \$2 respectiviely. The products are made using three processes: separating, blending, and mixing. One batch of Ocean Fresh requires 2 hours of separating, 1 hour of blending, and 4 hours of mixing. One batch of Summer Breeze requires 1 hour of separating and 2 hours of mixing. One batch of Lemon Zest requires 1 hour of blending and 1 hour of mixing. In a week it is possible to do 25 hours of separating, 30 hours of blending, and 40 hours of mixing. The company would like to determine how many batches of each product should be made to maximize weekly revenue.
    (a) Name variables and set-up a linear program that represents this situation.
    (b) Solve the problem using the simplex algorithm.
    (c) When revenue is maximized, how many hours of each process go unused?

19. Alice, Bob, and Cathy together make Xylophones, Yoyos, and Zippers. To make a Xylophone takes 4 hours of Alice's time, 2 hours of Bob's time, and 10 hours of Cathy's time. (So together it takes them 16 hours.) Similarly to make a Yoyo takes 9 hours for Alice, 3 hours for Bob, and 5 hours for Cathy; and to make a Zipper takes 3 hours, 1 hour, and 6 hours of their time respectively. Meanwhile, Alice has 120 hours available, Bob has 50 hours available, and Cathy has 400 hours available. Finally, Xylophones sell for \$12 each, Yoyos sell for \$8 each and Zippers sell for \$7 each.
    (a) Name variables and set-up a linear program to maximize revenue under these conditions.
    (b) Solve the program to determine how many Xylophones, Yoyos, and Zippers are made when revenue is maximized.
    (c) How many hours do Alice, Bob, and Cathy each have left over when revenue is maximized?

ANSWERS ON NEXT PAGE.

ANSWERS:

1. Min $z = -12$ at $(12/7, 12/7)$. Other corners: $(0,0), (4,0)$.

2. Max $z = -7$ at $(8,1)$. Other corners: $(9,0), (13,0), (12,2)$.

3. Max $z = 52$ at $(8,1)$. Other corners: $(1,1), (4,8), (7,8)$.

4. Min $z = 60/17$ at $(28/17, 12/17)$.
   Other corners: $(2,0), (4,0), (7,3)$.

5. Min $z = 0$ occurs at $(12,12)$. Other corners: $(8,4), (16,0)$.
   Note: the region is unbounded, but lines of constancy show that the minimum still exists.

6. Max $z = 40$ at $(10,0,2,0,38,0)$.

7. Max $z = 370$ at $(25,0,10,5,0,0)$

8. Min $z = -20$ at $(1,0,5,19,0,0)$.

9. Min $z = -16/3$ at $(0,0,8/3,2,0)$. Note: this is an example where choosing the column whose top entry is furthest from zero does not give the shortest path to the answer.

10. $z = 1312$ at $(0,303,100,0,213)$ for example.

11. $z = -2012$ at $(1000,2504,5006,19025,0,0)$.

12. $z = -7006$ at $(5002,2000,0,18011,0,10005)$.

13. $z = 5012$ at $(1000,2003,0,2,0,5015)$

14. (a) Max $z = 7$ at $(3,4)$.
    Other corner points: $(0,0), (0,5), (4,2), (4,0)$.
    (b) Max $z = 7$ at $(3,4,0,0,1)$.
    (c) There are two possibilites:
    First pivot in the x_2 column gives: $A(0,0), B(0,5), C(3,4)$.
    First pivot in x_1 column: $A(0,0), B(4,0), C(4,2), D(3,4)$.

15. Let x =# of dry martinis, y =# of medium martinis.

$$\begin{array}{lrcrcl}
\textbf{Maximize} & z = 8x & + & 10y & & \\
\text{subject to} & \frac{5}{3}x & + & \frac{3}{2}y & \leq & 60 \\
 & \frac{1}{3}x & + & \frac{1}{2}y & \leq & 16 \\
 & x \geq 0, & & y \geq 0 & &
\end{array}$$

Maximum Revenue is \$344 when 18 dry martinis and 20 medium martinis are sold. Other corner points: $(0,0), (36,0), (0,32)$.

16. Let x_1 =number of Widgets made; let x_2 =number of Gadgets made; and let x_3=number of Gizmos made.

$$\begin{array}{lcrcrcrcl}
\textbf{Maximize} & z = & 2x_1 & + & 4x_2 & + & x_3 & & \\
\text{subject to} & & 3x_1 & + & 7x_2 & + & 3x_3 & \leq & 56 \\
 & & 2x_1 & & & + & 2x_3 & \leq & 16 \\
 & & 4x_1 & + & x_2 & + & 5x_3 & \leq & 33 \\
 & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 & &
\end{array}$$

The maximum revenue is \$34 when 7 widgets, 5 gadgets, and 0 gizmos are made. When revenue is maximized there will be 0 pulleys, 2 wedges, and 0 levers remaining.

17. (a) Let x_1 be the number of beds,
    let x_2 be the number of chairs, and
    let x_3 be the number of couches made.

$$\begin{array}{lrcrcrcl}
\textbf{Maximize} & z = 60x_1 & + & 10x_2 & + & 40x_3 & & \\
\text{subject to} & x_1 & + & x_2 & + & 2x_3 & \leq & 100 \\
 & 3x_1 & + & 5x_2 & + & x_3 & \leq & 90 \\
 & 3x_1 & + & 7x_2 & + & x_3 & \leq & 120 \\
 & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 & &
\end{array}$$

    (b) Max profit = \$2640.
    (c) Profit is maximized when 16 beds, 0 chairs, and 42 couches are made.
    (d) When profit is maximized, there are 0 hours of labor, 0 meters of lumber, and 30 meters of cloth unused.

18. (a) Let x_1 =number of batches of Ocean Fresh;
    let x_2 =number of batches of Summer Breeze; and
    let x_3=number of batches of Lemon Zest.

$$\begin{array}{lcrcrcrcl}
\textbf{Maximize} & z = & x_1 & + & 3x_2 & + & 2x_3 & & \\
\text{subject to} & & 2x_1 & + & x_2 & & & \leq & 25 \\
 & & x_1 & & & + & x_3 & \leq & 30 \\
 & & 4x_1 & + & 2x_2 & + & x_3 & \leq & 40 \\
 & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 & &
\end{array}$$

    (b) The maximum revenue is \$75 when 0 batches of Ocean Fresh, 5 batches of Summer Breeze, and 30 batches of Lemon Zest are made.
    (c) When revenue is maximized there will be 20 hours of separating, 0 hours of blending, and 0 hours of mixing remaining.

19. (a) Let x_1 =# of Xylophones; let x_2 =# of Yoyos; and
    let x_3 =# of Zippers.

$$\begin{array}{lcrcrcrcl}
\textbf{Maximize} & z = & 12x_1 & + & 8x_2 & + & 7x_3 & & \\
\text{subject to} & & 4x_1 & + & 9x_2 & + & 3x_3 & \leq & 120 \\
 & & 2x_1 & + & 3x_2 & + & x_3 & \leq & 50 \\
 & & 10x_1 & + & 5x_2 & + & 6x_3 & \leq & 400 \\
 & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0 & &
\end{array}$$

    (b) Maximum revenue of \$320 occurs when 15 Xylophones, 0 Yoyos, and 20 Zippers are made.
    (c) When revenue is maximized Alice and Bob have 0 hours left over, while Cathy has 130 hours left over.