JOHN ABBOTT COLLEGE

MATHEMATICS DEPARTMENT

FINAL EXAM

Date:	January 8, 2021	**Name**:____________________________
Course:	Calculus III Math 201-DDB-05	
Instructor:	Frank Lo Vasco	**Student Number**:___________________
Time:	2 hours	

Note:

1) Check that this test contains 13 pages (counting the cover page).
2) Write your solutions in the space provided. Your solutions must be displayed neatly (not crowded) and all supporting work must be shown.
3) If the space provided is not sufficient, continue your work on the reverse side.
4) The use of a scientific calculator **is permitted but is not really necessary**

(12) 1. **Evaluate** the following integrals:

(Change coordinates or the order of integration as appropriate.)

i) $\int_0^8 \int_{\sqrt[3]{y}}^2 \frac{1}{1+x^4}\,dx\,dy$

ii) $\int_0^3 \int_x^{\sqrt{18-x^2}} \sin(x^2 + y^2)\, dydx$

(6) 2. i) Sketch the solid region S in the first octant bounded by the coordinate planes and the surfaces $z = 4 - x^2$ and $x + y = 2$;

ii) Set up (**but do not evaluate**) a triple integral needed to find its Volume.

(10) 3. Given $I = \int_0^2 \int_0^{\sqrt{4-x^2}} \int_{x^2+y^2}^{4} \sqrt{x^2+y^2}\ dzdydx$

a) Rewrite I as $\iiint_S \sqrt{x^2+y^2}\ dxdzdy$

b) Rewrite I using *Cylindrical coordinates*

c) Rewrite I using *Spherical coordinates*

(5) 4. **Set up** a triple integral in cylindrical coordinates to find find the volume of that part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 2x$.

(12) 5. i) Sketch the space curve defined by $r(t) = \langle 2\cos t,\ 3t\ ,\ 2\sin t \rangle$

ii) Find: *a*) the velocity, the acceleration and speed of a particle moving along this curve.

b) the equation of the tangent line to this curve at $t = \pi/3$

c) the curvature at $t = \pi/3$

(14)6. Let $P\ (-4,-2,1)$ be a point on the level surface S defined by:

$$F(x,y,z) = x - y^3 - 2z^2 = 2$$

Find:

i) the equation of the tangent plane to S at the point P.

ii) the directional derivative of F at P in the direction of $v = (3,6,-2)$

iii) the maximum rate of change in F at P

iv) the tangent line to the curvew C at P, where C is the curve of intersection of the level surface S and the plane $2x - 3y - z = -3$

(9) 7. Sketch and name the following surfaces:

i) $z = \sqrt{x^2 - 4y^2 + 1}$

ii) $z = r^2 + 4$

iii) $\rho = 4\cos\phi$

(6)8. Show that $f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

is continuous for all $(x,y) \in R^2$.

(20) 9. *a*) Calculate $\frac{\partial^2 z}{\partial y \partial x}$ for the function $z = e^{3xy^2} + 4x^3 - y^3 \ln x$

b) Find $\frac{\partial z}{\partial x}$ Given $z = x^2 \sin(xy^3) + \ln(y^2 - 2x^3)$

c) Find $\frac{\partial z}{\partial x}$ Given $z = f(x,y)$ is implicitly defined by: $z = e^x \cos(y^2 + z^2) + 3x^2\, y\, z$

d) Find $\frac{\partial z}{\partial x}$ Given $z = u^2 v + 3ue^v$ and $u = 3x - 2y^2$ and $v = x^3 \tan y$

(6)10. Find and classify the critical points of $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$