1. (3 marks) Solve the following system of equations for x only, using Cramer's Rule.

$$\begin{array}{rcrcrcr} 2x & + & 3y & & & = & 15 \\ x & + & 2y & + & 5z & = & 1 \\ -3x & + & y & + & 8z & = & 0 \end{array}$$

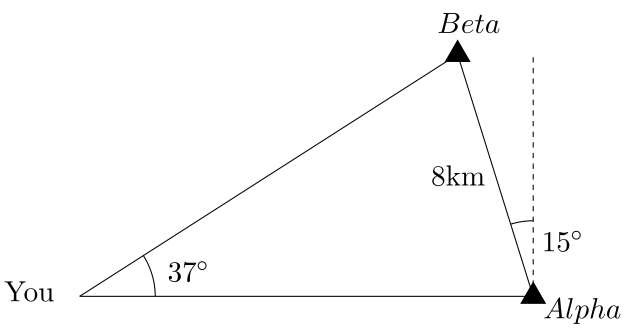
2. (3 marks) You are lost in the mountains and you are trying to determine your location with your map and compass.

You can see mountaintops *Alpha* and *Beta* from your current position.

Using your compass, you can determine that mountaintop *Alpha* is directly east of your current position, while mountaintop *Beta* has a bearing of 37° north of east.

You can see on your map that mountaintop *Beta* is 8 kilometers away from mountaintop *Alpha*, with a bearing of 15° west of north.

What is your distance from mountaintops *Alpha* and *Beta* ?



3. (6 marks) Solve the following for x. Give simplified exact answers.

(a) $25^{3x-1} - 5^{2-x} = 0$

(b) $\log_{12}(2x-3) + \log_{12}(x+5) - \log_{12}(x-1) = 1$

4. (4 marks) Perform the following operations and write your answer in **rectangular form**.

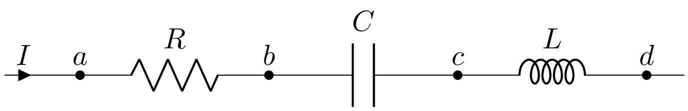
(a) $(j + j^2 + j^3 + j^4 + j^5)^{100}$

(b) $\dfrac{j(2+j)}{1+4j}$

5. (3 marks) Use De Moivre's theorem to evaluate $\left(-2 + 2\sqrt{3}j\right)^5$. Write your answer in **polar form**, with an angle $0° \le \theta \le 360°$.

6. (4 marks) Consider the series RCL electrical circuit satisfying the following:

- The current has a frequency of $f = 15.0$ Hz;
- The resistance is $R = 35.0\ \Omega$;
- The inductance is $L = 0.400$ H;
- The capacitance is $C = 86.4\ \mu$F;



Determine the following:

(a) the total impedence Z and its magnitude $|Z|$;

(b) whether the voltage leads or lags the current, and by what angle.

7. (12 marks) Evaluate the following limits:

(a) $\displaystyle\lim_{x\to -3} \frac{x^2+4x+3}{x^3+27}$

(b) $\displaystyle\lim_{x\to 4^+} \frac{3+x}{16-x^2}$

(c) $\displaystyle\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$

(d) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{3x^2+4x-1}}{3-x}$

8. (4 marks) Use the **limit definition of derivative** to find $f'(x)$ where $f(x) = \sqrt{x+2}$.

9. (5 marks) Consider the function $f(x) = \dfrac{\cos x}{2+\sin x}$.

(a) Find $f'(x)$ and simplify your answer.

(b) Find values of x in $[0, 2\pi)$ for which the graph of f has a horizontal tangent.

10. (12 marks) Find the derivative of the following functions (**You do not need to simplify your answers**):

(a) $y = 7x^7 + \sqrt[7]{x} + \csc(x) + 7^x + \log_7(x) + 2020\pi$

(b) $y = (3x^2+1)^9 \tan(x^3+7)$

(c) $y = 5\sec^9(x^6+e^x)$

(d) $y = \ln\left(\dfrac{\cos^6(x)\,\sqrt[5]{5x^4+x}}{(x+7)^8\cot(x^5)}\right)$ *(Hint: begin by simplifying the logarithm)*

11. (3 marks) Use implicit differentiation to find y', given that $x^2 + y^3 = x^3y^2 + 1$.

12. (4 marks) Sketch the graph of a function f that satisfies the following limits:

$\displaystyle\lim_{x\to-\infty} f(x) = \infty$ $\displaystyle\lim_{x\to-1} f(x) = 2$ $\displaystyle\lim_{x\to 2^-} f(x) = 5$ $\displaystyle\lim_{x\to 4^-} f(x) = \infty$ $\displaystyle\lim_{x\to\infty} f(x) = -3$

$f(-1)$ is undefined $\displaystyle\lim_{x\to 2^+} f(x) = -2$ $\displaystyle\lim_{x\to 4^+} f(x) = -\infty$

$f(2) = 5$

Answers

1. -3

2. to Alpha: 12.32 km. to Beta: 12.84 km.

3. (a) $x = 4/7$

(b) $x = 3$

4. (a) 1

(b) $\frac{7}{17} + \frac{6}{17}j$

5. $1024\angle 240°$

6. (a) $|Z| = 92.02\Omega$

(b) Voltage lags current by 67.6°

7. (a) $-2/27$

(b) $-\infty$

(c) $1/2$

(d) $\sqrt{3}$

8. $f'(x) = \dfrac{1}{2\sqrt{x+2}}$

9. (a) $f'(x) = \dfrac{-2\sin(x) - 1}{(2+\sin(x))^2}$

(b) $x = \dfrac{7\pi}{6}$, $x = \dfrac{11\pi}{6}$

10. (a) $y' = 49x^6 + \frac{1}{7}x^{-6/7} - \csc(x)\cot(x) + 7^x \ln 7 + \dfrac{1}{x \cdot \ln 7}$

(b) $y' = 9\left(3x^2+1\right)^8 \cdot 6x \cdot \tan\left(x^3+7\right) + \left(3x^2+1\right)^9 \cdot \sec^2\left(x^3+7\right) \cdot 3x^2$

(c) $y' = 45\sec^8\left(x^6+e^x\right) \cdot \sec\left(x^6+e^x\right)\tan\left(x^6+e^x\right) \cdot \left(6x^5+e^x\right)$

(d) $y' = -6\tan x + \dfrac{20x^3+1}{5\left(5x^4+x\right)} - \dfrac{8}{x+7} + \dfrac{\csc^2\left(x^5\right) \cdot 5x^4}{\cot\left(x^5\right)}$

11. $y' = \dfrac{3x^2y^2 - 2x}{3y^2 - 2x^3y}$

12. Hole at (-1,2). Jump at $x = 2$ from $y = 5$ (full) to $y = -2$ (empty). Vertical asymptote at $x = 4$. Horizontal asymptote at $y = -3$ on the right side.