(5) 1. Let $f(x) = \sqrt[3]{8+x}$.

(a) Use the Binomial Theorem to find the Maclaurin series for f. Write your answer in Σ notation.

(b) Find the radius of convergence for this series.

(6) 2. Let $g(x) = \displaystyle\int_0^x t\cos\sqrt{t}\,dt$

(a) Find the Maclaurin series for $g(x)$; express your answer in $\sum$ form.

(b) Find the radius of convergence for this series.

(c) Find $g(\frac{1}{2})$ correct to 4 decimal places.

(6) 3. For the function $f(x) = \sqrt{x}$:

(a) Find the third degree polynomial $T_3(x)$ centered at $a = 1$.

(b) Use $T_3(x)$ to approximate $f(1/2)$.

(c) Use Taylor's Inequality to estimate the maximum error of your approximation.

(8) 4. Given the curve $\mathcal{C}$ having parametric equations: $x = t^2 + 2t$, $y = 2t - t^2$

(a) Find the x and y intercepts.

(b) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. Simplify your answers.

(c) Find the points on $\mathcal{C}$ where the tangent line is vertical or horizontal.

(d) Sketch the graph of $\mathcal{C}$ for $-2 \le t \le 2$, showing the orientation of the curve.

(e) Set up, **but do not evaluate**, an integral for the area of the region bounded by $\mathcal{C}$ and the y-axis.

(f) Set up, **but do not evaluate**, an integral for the arc length of $\mathcal{C}$ on $-2 \le t \le 2$.

(8) 5. Given the polar curves $r_1 = 4\cos\theta$ and $r_2 = 3\sec\theta$, do the following:

(a) Sketch both graphs on the same axes.

(b) Find all the points of intersection for $\theta \in [0, 2\pi)$. Give your answer(s) in Cartesian coordinates.

(c) r_1 encloses a region and r_2 cuts this region into two parts. Set up, **but do not evaluate,** an integral expression for the area of the smaller of these two parts.

(d) Set up, **but do not evaluate,** an integral for the length of r_1.

(10) 6. Let $\mathcal{C}$ be the space curve represented by $\mathbf{r}(t) = \langle e^t\cos t, e^t\sin t, e^t\rangle$.

(a) Find parametric equations for the tangent line to $\mathcal{C}$ at $P(1, 0, 1)$.

(b) Find an equation (in $ax + by + cz = d$ form) of the normal plane of $\mathcal{C}$ at $P(1, 0, 1)$.

(c) Find the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$.

(d) Find the curvature $\kappa(t)$.

(e) Find the tangential and normal components of the acceleration vector (a_T and a_N) as functions of t.

(6) 7. Sketch and name the following surfaces:

(a) $z - x^2 = 0$

(b) $z^2 - r^2 + 4 = 0$

(c) $\rho = \csc\phi \cot\phi$

(4) 8. Find the limit if it exists or show that it does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2\cos^2(y)}{\sqrt{x^4+3y^4}}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2(y)}{\sqrt{x^4+3y^4}}$

(4) 9. Let $f(x,y) = \sqrt{xy}$.

(a) Explain why f is differentiable at the point $(3,1)$.

(b) Find the linearization of f at the point $(3,1)$.

(3) 10. Let $z = yf(x^2 - y^2)$ where f is differentiable. Show that $y\dfrac{\partial z}{\partial x} + x\dfrac{\partial z}{\partial y} = \dfrac{xz}{y}$.

(3) 11. If $z = f(u,v)$ has continuous second-order partial derivatives and $u = 2x + y$ and $v = x - y$, find:

(a) z_y in terms of z_u and z_v.

(b) z_{xx} in terms of z_{uu}, z_{uv}, and z_{vv}.

(3) 12. Determine the set of points at which

$$f(x,y) = \begin{cases} \frac{2xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is continuous. Justify your answer.

(5) 13. Given the surface $\mathcal{S} : f(x,y,z) = 4x^2 + 4y^2 + 7z^2 = 15$ and $P(1,1,1)$, find:

(a) the directional derivative of f at the point P in the direction of $\mathbf{v} = \langle 2, 3, 6\rangle$.

(b) the equation of the tangent plane at P.

(c) the minimum rate of change in f at P.

(4) 14. Find and classify the critical points of $f(x,y) = x^3 + y^2 + 2xy - 4x - 3y + 5$.

(4) 15. Use the method of Lagrange multipliers to find the maximum and minimum of $f(x,y) = x^2 + y^2 + 4x - 4y + 3$ on the circle $x^2 + y^2 = 2$.

(9) 16. Evaluate

(a) $\displaystyle\int_0^4 \int_{x/2}^2 e^{y^2}\, dydx$

(b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \cos(x^2+y^2+4)\, dydx$

(c) $\displaystyle\iiint_{\mathcal{H}} \sqrt{x^2+y^2+z^2}\, dV$ where $\mathcal{H}$ is the region $x^2+y^2+z^2 \leq 1$, $z \leq 0$.

(6) 17. Let S be the solid bounded above by the hemisphere $z = \sqrt{4-x^2-y^2}$, below by the xy-plane, and laterally by the cylinder $x^2+y^2=1$. Set up (**do not evaluate**) triple integrals needed to find its volume in

(a) cartesian coordinates

(b) cylindrical coordinates

(c) spherical coordinates

(6) 18. Rewrite the integral $\displaystyle\int_0^1 \int_{2x}^2 \int_0^{4-y^2} dzdydx$ in the order

(a) $dxdzdy$

(b) $dydzdx$

Do not evaluate. Partial credit will be awarded for clear and detailed solutions. You may wish to make two (or three) dimensional sketches.

ANSWERS

1. (a) $\displaystyle 2\left(1+\frac{x}{24}+\sum_{n=2}^{\infty}\frac{(-1)^{n+1}2\cdot 5\cdots(3n-4)}{24^n n!}x^n\right)$ (b) 8

2. (a) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+2)(2n)!}x^{n+2}$ (b) ∞ (c) 0.1048

3. (a) $1+\frac{1}{2}(x-1)-\frac{1}{8}(x-1)^2+\frac{1}{16}(x-1)^3$ (b) 0.7109... (c) 0.0276...

4. (a) $(0,0),(0,-8),(8,0)$ (b) $\dfrac{1-t}{1+t}$, $\dfrac{-1}{(t+1)^3}$ (c) $(3,1)$ (horiz.), $(-1,-3)$ (vert.) (d) (e) $\displaystyle -\int_{-2}^0 (t^2+2t)(2-2t)\, dt$ (f) $\int_{-2}^2 \sqrt{(2t+2)^2+(2-2t)^2}\, dt$

5. (a) (b) $(3, \pm\sqrt{3})$ (c) $\displaystyle\int_0^{\pi/6} [(4\cos\theta)^2 - (3\sec\theta)^2]\, dt$ (d) $\displaystyle\int_0^{\pi} \sqrt{(-4\sin\theta)^2+(4\cos\theta)^2}\, d\theta$

6. (a) $\mathbf{x} = \langle 1,0,1\rangle + s\langle 1,1,1\rangle$ (b) $x+y+z=2$ (c) $\mathbf{T} = \frac{1}{\sqrt{3}}\langle \cos t - \sin t, \sin t + \cos t, 1\rangle$ $\mathbf{N} = \frac{1}{\sqrt{2}}\langle -\cos t - \sin t, \cos t - \sin t, 0\rangle$ (d) $\kappa = \frac{\sqrt{2}}{3e^t}$ (e) $a_T = v' = \sqrt{3}e^t$, $a_N = \kappa v^2 = \sqrt{2}e^t$

7. (a) parabolic cylinder (b) hyperboloid of one sheet (c) circular paraboloid
8. (a) DNE (b) 0 (Squeeze)
9. (a) Both partials exist near $(3,1)$ and are cts. at $(3,1)$ (b) $L(x,y) = \frac{x-3}{2\sqrt{3}} + \frac{\sqrt{3}(y-1)}{2} + \sqrt{3}$
10.
11. (a) $z_u - z_v$ (b) $4z_{uu} + 4z_{uv} + z_{vv}$
12. f is cts. everywhere (on $\mathbb{R}^2$).
13. (a) $\frac{124}{7}$ (b) $4x + 4y + 7z = 15$ (c) -18
14. $(-1/3, 11/6)$ (saddle), $(1, 1/2)$ (loc. min.)
15. max of 13 at $(1,-1)$, min of -3 at $(-1,1)$
16. (a) $e^4 - 1$ (b) $\frac{\pi}{4}(\sin 5 - \sin 4)$ (c) $\frac{\pi}{2}$
17. (a) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^{\sqrt{4-x^2-y^2}} dzdydx$ (b) $\int_0^1\int_0^{2\pi}\int_0^{\sqrt{4-r^2}} r\,dzd\theta dr$ (c) $\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc\phi} \rho^2 \sin\phi\, d\rho d\phi d\theta$
18. (a) $\int_0^2\int_0^{4-y^2}\int_0^{y/2} dxdzdy$ (b) $\int_0^1\int_0^{4-4x^2}\int_{2x}^{\sqrt{4-z}} dydzdx$