Evaluate the following integrals

a. $\int \frac{2}{x\ln(x)}\,dx$

b. $\int \frac{(x-1)^2 + x^{3/2}\,e^x}{\sqrt{x^3}}\,dx$

c. $\int_{-1}^{2} \left(|1-x| + x\right)dx$

d. $\int \csc(4x+1)\cot(4x+1)\,dx$

e. $\int_{1}^{2} \frac{12}{t^3 - 6t^2}\,dt$

f. $\int 8x\sec^2(3x)\,dx$

g. $\int \frac{x^2 + 5x + 4}{x+2}\,dx$

2. Suppose $\int_{-2}^{3} \left(1 - 2f(x)\right)dx = -1$ and $\int_{-5}^{3} f(x)\,dx = 10$, calculate $\int_{-5}^{-2} f(x)\,dx$

3. Find values of a and b which satisfy the equation $\int_{1}^{a} f(x)\,dx - \int_{b}^{4} f(x)\,dx = \int_{4}^{10} f(x)\,dx$

4. Consider the functions $f(x) = x^2 + 1$ and $g(x) = x^3 + 1$.
   a. Find the points of intersection of the graphs of f and g.
   b. Calculate the area of the region completely enclosed by the graphs of f and g.

5. Given the demand function $p(x) = \frac{12}{x+3}$ and the supply function $p(x) = x + 2$
   a. Find the equilibrium point.
   b. Sketch and identify the regions representing the consumer and producer surpluses.

c. Calculate the consumer surplus.

6. a. Given that $\int_1^2 2xe^{x^2}$, use the trapezoid rule with $n=2$ to obtain an approximation for the area under the graph between $1 \le x \le 2$. Your answer should be accurate to 4 decimal places.

   b. Use the substitution rule to obtain an exact value for the area accurate to 4 decimal places.

7. Find the function y that satisfies the differential equation $y' = 6y^2x$ with $y(1) = \frac{1}{9}$.
8. A Tech company has upgraded its computer infrastructure by purchasing several new computers for a total value V of $8000. The rate of depreciation value in dollars at time t in years is proportional to the square root of its value V. The computers will be worth $5000 three years later. What is the value of the computers after 5 years?
9. Evaluate the limits.

   a. $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$

   b. $\lim_{x \to -\infty} \frac{e^{-3x}}{5+e^{-2x}}$

10. Evaluate the improper integrals.

   a. $\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$

   b. $\int_1^5 \frac{7}{(x-1)}\,dx$

11. Find a formula for the n^{th} term of the sequence $\left\{ \frac{2}{5}, \frac{4}{15}, \frac{8}{45}, \frac{16}{135}, \ldots \right\}$
12. Determine whether the following sequence converges or diverges. If the sequence converges, find its limit. If sequence diverges, explain why.

$$a_n = \frac{(-1)^n (1+7n)}{(1-2n)}$$

13. Given $a_n = \frac{5(2n)!}{3n\,(2n-1)!}$

   a. Does the sequence converge? Justify your answer

b. Does $\sum_{n=1}^{\infty} a_n$ converge?

14. Use the word Must, Might or Cannot which best completes the statement below:

If the series $\sum_{n=1}^{\infty} a_n$ converges , then the sequence -------------------------------converge to zero.

15. Determine whether the following series converge or diverge. Identify which test you are using. If the convergent series is geometric or telescoping, find its sum.

a. $$\sum_{n=1}^{\infty} \frac{(n+1)^5\, 2^n}{5^{n+1}}$$

b. $$\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n-1}}$$

c. $$\sum_{n=3}^{\infty} \frac{1}{n^2-n-2}$$

d. $$\sum_{n=1}^{\infty} \frac{n^2\,(n+1)^2}{10n^4}$$

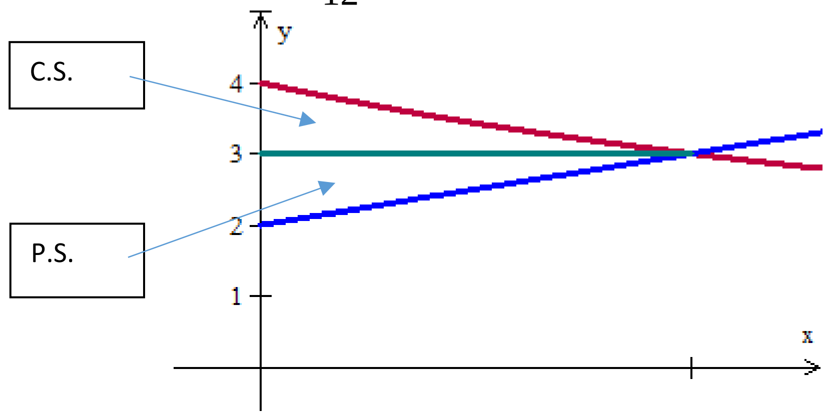
16. Karen is planning a trip to Canada to visit her friend in two years time. She makes an itinerary for her holiday and expects that the trip will cost $4000 USD. How much must she save every month if her savings account earns an interest rate of 3% per annum compounded monthly?

Answers:

1) a) $2\ln|\ln(x)|+c$; b) $\frac{2}{3}x^{3/2}-4\sqrt{x}-2x^{-1/2}+e^{x}+c$; c) 4; d) $\frac{-1}{4}\csc(4x+1)+c$

e) $\ln\left(\frac{2}{5}\right)^{1/3}-1=-1.305$; f) $\frac{8x}{3}\tan(3x)-\frac{8}{9}\ln|\sec(3x)|+c$; g) $\frac{x^2}{2}+3x-2\ln|x+2|+c$

2) 7; 3) $a=10, b=1$; 4) a) $x=0, x=1$; b) $\frac{1}{12}$; 5) a) $(1,3)$; b)



c) $\$0.452$; 6) a) 70.1889; b) 51.8799; 7) $y=\frac{-1}{3x^2-12}$; 8) $\$3389.88$

9) a) $\frac{1}{2}$; b) ∞; 10) a) $\frac{e-1}{e}$; b) Div. ; 11) $\frac{2^n}{5\,3^{n-1}}$

12) *Div.* 13) a) *conv.to* $\frac{10}{3}$; b) Div. by Divergence T.; 14) Must; 15) a) Conv. By Ratio T.

b) Conv. By Geometric ST. and sum= $\frac{15}{2}$; c) Conv. By Telescoping ST. and sum= $\frac{11}{18}$; d) Div. by Divergence T.; 16) $\$161.5$