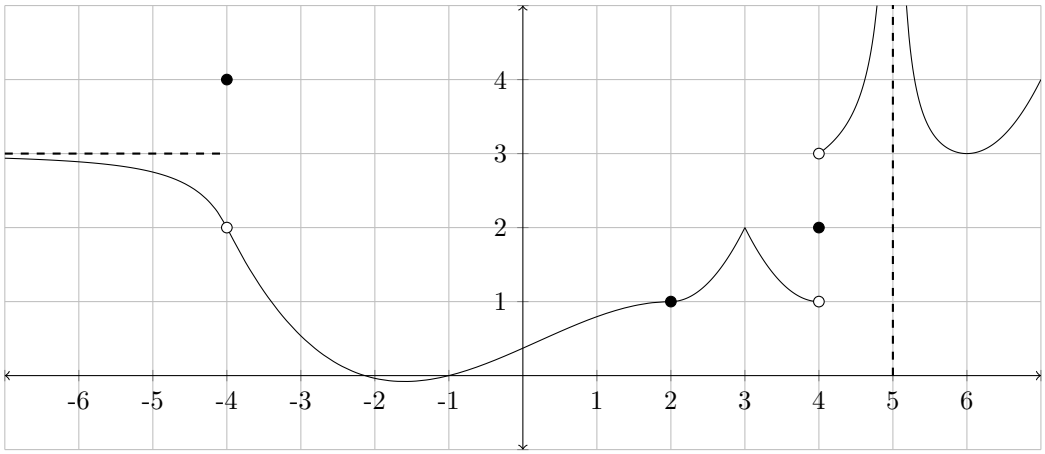
1. Given the graph of f below, determine each of the following. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.



(a) $f(-4) =$

(b) $f(5) =$

(c) $\lim_{x\to -4} f(x) =$

(d) $\lim_{x\to 2} f(x) =$

(e) $\lim_{x\to 4^-} f(x) =$

(f) $\lim_{x\to 4} f(x) =$

(g) $\lim_{x\to 5^+} f(x) =$

(h) $\lim_{x\to -\infty} f(x) =$

(i) $\lim_{x\to \infty} f(x) =$

(j) $f'(6) =$

(k) $\lim_{x\to 2} (f(x) + 2x) =$

(l) At what x values is the graph of $f(x)$ not differentiable? **Provide an explanation for each x value.**

2. Evaluate the following limits.

(a) $\lim_{x\to -1} \dfrac{x+1}{\sqrt{x+5}-2}$

(b) $\lim_{w\to 2^+} \dfrac{|2-w|}{3w^2-5w-2}$

(c) $\lim_{x\to 5^-} \dfrac{x^2-3x-10}{x^2-10x+25}$

(d) $\lim_{x\to \infty} \dfrac{2x^2(x-5)^2}{3(x+1)(2x-5)^3}$

3. Find the values of A and B such that $g(x)$ has a horizontal asymptote of $y = 3$ and a vertical asymptote of $x = 5$.

$$g(x) = \frac{Ax^2+3x+10}{2x(x-B)}$$

4. Determine where $h(x)$ is **discontinuous** using the definition of continuity.

$$h(x) = \begin{cases} \dfrac{x+9}{(x-2)(x+4)} & x \le 1 \\ -2 & 1 < x < 8 \\ \sqrt{3x+1} & 8 \le x \end{cases}$$

5. Complete the following sentences with one of the words **MUST**, **MIGHT**, or **CANNOT**, as appropriate:

(a) If $f(x) = \frac{P(x)}{Q(x)}$ and $Q(2) = 0$ then $f(x)$ _____ have a vertical asymptote of $x = 2$.

(b) A function $f(x)$ _____ have 3 different horizontal asymptotes.

(c) A function $f(x)$ _____ have 3 different vertical asymptotes.

6. Given $f(x) = \dfrac{5}{x+1}$

(a) Use the **definition of the derivative** to find $f'(x)$.

(b) Find the slope of the tangent line of f at $x = 3$.

7. Find y' for each of the following. **Do not simplify your answers**.

(a) $y = \dfrac{5x^2}{3} + \log_5(x) - \sec(2x) + \dfrac{2}{\sqrt[7]{x}} + \pi^{512+e}$

(b) $y = 5xe^{2x} + \sqrt{\tan(x)}$

(c) $y = (1 + \sqrt{x})^{3x}$

(d) $y = \dfrac{5^{3x}}{\cos(2x)}$

(e) $y = \sin\left(\ln(x^2 + x) - 7x\right)$

(f) $y = \ln\left(\dfrac{(2x+4)^3 e^{4x}}{\cot^5(x)}\right)$

8. Find the 2nd derivative of $y = \dfrac{6x^4 + 5x + 4 + 2x^3 e^x}{2x}$.

9. Find the 2019^{th} derivative of $y = \sin(2x+1) + 5x^6 + 20x^{100}$.

10. Find the equation of the tangent line to the curve $(x+y)^2 = 7x - 3xy$ at the point (1,1).

11. Use the second derivative test to find the local extrema of $g(x) = 3x^4 + 8x^3 + 6x^2 - 1$. If the test fails, simply state this.

12. If $f(x) = 24x^{1/3} - 3x^{4/3}$.

(a) Find both absolute extrema (if they exist) for $f(x)$ in the interval [-1,2]

(b) What (if anything) about your answer in part a would change if the interval was (-1,2) instead?

13. Given that $F(x) = \dfrac{(x+2)^2}{(x-2)^2}$, $\quad F'(x) = \dfrac{-8x-16}{(x-2)^3}$, $\quad$ and $F''(x) = \dfrac{16x+64}{(x-2)^4}$.

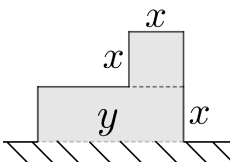
Determine the the domain of F, all asymptotes, all intercepts, the intervals on which F is increasing and decreasing, and on which F is concave up and concave down, as well as all local extrema and the inflection points of F. Then sketch F.

14. The JAC foundation is planning to raise money by holding a dinner gala in the new art installation. When tickets are sold at \$40 per plate, 60 guests attend. For each decrease in price of \$1 per plate, five more guests will attend.

(a) What ticket price will maximize the funds raised by this event?

(b) If they are hoping to raise \$4000, will that price be enough to meet their goal?

15. You are tasked with designing the new John Abbott 2D art installation inspired by a Tetris L block. The goal is to use the smallest amount of material possible, since it'll sit on a platform you do not need to construct the base. Minimize the perimeter (excluding the base) of the following shape if it has an area of 108.



16. Assume that the demand equation of a product is $x = -p^2 + p + 35$.

(a) Find the price elasticity of demand function.

(b) When the price is \$ 5, what will happen to the quantity demanded if the price increases by 2 %?

(c) If the price is instead set at \$ 2, should the price be increased or decreased in order to increase revenue? Explain.

ANSWERS:

1. Graph provided

(a) 4

(b) dne

(c) 2

(d) 1

(e) 1

(f) dne

(g) ∞

(h) 3

(i) ∞

(j) 0

(k) 5

(l) $x = -4$ discntns (removable), $x = 3$ cusp, $x = 4$ discntns (jump), $x = 5$ discntns (VA)

2. (a) 4

(b) $\frac{1}{7}$

(c) $-\infty$

(d) $\frac{1}{12}$

3. $A = 6, B = 5$

4. Discntns ar $x = -4, 8$

5. (a) If $f(x) = \frac{P(x)}{Q(x)}$ and $Q(2) = 0$ then **MIGHT** have a vertical asymptote of $x = 2$.

(b) A function **CANNOT** have 3 different horizontal asymptotes.

(c) A function **MIGHT** have 3 different vertical asymptotes.

6. (a) $f(x) = \dfrac{-5}{(x+1)^2}$

(b) $f(x) = \dfrac{-5}{16}$

7. (a) $y' = \dfrac{10}{3}x + \dfrac{1}{x\ln(5)} - \sec(2x)\tan(2x)(2) - \dfrac{2}{7}x^{-8/7}$

(b) $y' = 5e^{2x} + 5xe^{2x}(2) + \dfrac{1}{2}(\tan x)^{-1/2}\sec^2 x$

(c) $y' = \left[3\ln(1+\sqrt{x}) + \dfrac{3x}{1+\sqrt{x}}(\dfrac{1}{2}x^{-1/2}\right](1+\sqrt{x})^{3x}$

(d) $y' = \dfrac{\ln(5)5^{3x}(3)\cos(2x) - 5^{3x}(-\sin(2x))2}{(\cos^2(2x))^2}$

(e) $y' = \cos(\ln(x^2+x) - 7x)(\dfrac{2x+1}{x^2+x} - 7)$

(f) $y' = \dfrac{3}{2x+4}(2) + 4 - \dfrac{5}{\cot x}(-\csc^2 x)$

8. $y'' = 18x + 4x^{-3} + 2e^x + 4xe^x + x^2e^x$

9. $y^{(2019)} = -\cos(2x+1)\cdot 2^{2019}$

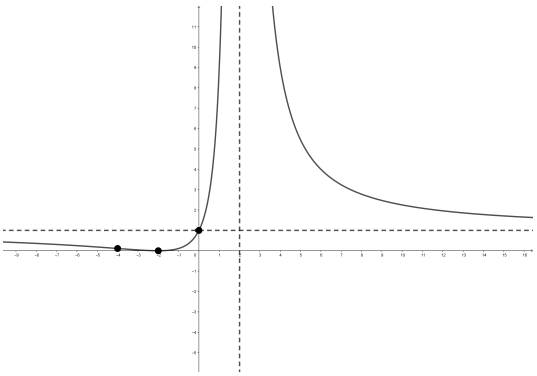
10. $y = 1$

11. local min at $x = 0$ test fails at $x = -1$

12. (a) Max $18\sqrt[3]{2}$ Min -27

(b) No max or min

13. Graph provided below



14. (a) \$ 26

(b) Nope

15. Smallest perimeter is 36 (at $x = 6$)

16. (a) $E(p) = \dfrac{-p(-2p+1)}{-p^2+p+35}$

(b) demand deceases by 3 %

(c) increase price (since inelastic)