

## Multiple choice

For each of the following questions, there is **only one correct answer**. Circle your choice. If two choices are selected for the same question, **no marks will be awarded**.

- The line  $y = 2x + 1$  has the same slope but a different  $y$ -intercept as which one of these lines?
  - $2y = x$
  - $x = \frac{y-1}{2}$
  - $y = 2$
  - $x = \frac{y-4}{2}$
- Which of these functions has a domain of  $(1, \infty)$ ?
  - $y = \frac{1}{\sqrt{x-1}}$
  - $y = \frac{1}{(x-1)^2}$
  - $y = \sqrt{x-1}$
  - $y = \frac{1}{x-1}$
- Suppose  $ax^2 + bx + c = 0$  has exactly one solution. Then:
  - $b^2 - 4ac > 0$
  - $b^2 - 4ac < 0$
  - $b^2 - 4ac = 0$
  - None of the above.
- Given that  $f(g(x)) = (x + 3)^2 - 1$ , then  $f$  and  $g$  can be:
  - $f(x) = x + 3, g(x) = x^2 - 1$
  - $f(x) = x^2 + 3, g(x) = x - 1$
  - $f(x) = x^2 - 1, g(x) = x + 3$
  - $f(x) = (x + 3)^2, g(x) = x - 1$
- Let  $\vec{u} = \langle 3, 2 \rangle$  and  $\vec{v} = \langle -2, 1 \rangle$ . Which of the following vectors is the longest?
  - $\vec{u}$
  - $\vec{v}$
  - $\vec{u} + \vec{v}$
  - $\vec{u} - \vec{v}$
- If a ball is thrown upwards at 29.4 metres per second from the top of a building that is 80 metres high, the height of the ball is given by  $h(t) = 80 + 29.4t - 4.9t^2$  where  $t$  is the number of seconds after the ball is thrown. What is the ball's maximum height?
  - 80m
  - 212.3m
  - 124.1m
  - 113.1m
- True or False:  $f(x) = \frac{(x-3)(x+2)}{(x+1)(x-3)}$  and  $g(x) = \frac{x+2}{x+1}$  have the same domain.
  - T
  - F
- The domain of  $f(x) = \sqrt{\frac{(x-1)(x+3)}{x(x+1)}} + 3x + 1$  is
  - $[-3, -1) \cup (0, 1]$
  - $[-3, -1) \cup (0, 1) \cup (1, +\infty)$
  - $(-\infty, -3) \cup (0, 1) \cup (1, +\infty)$
  - $(-\infty, -3] \cup (-1, 0) \cup [1, +\infty)$
- The graph of  $y = \log(x + 1) - 2$  has
  - a vertical asymptote at  $x = -2$
  - a vertical asymptote at  $x = -1$
  - a horizontal asymptote at  $y = -2$
  - a horizontal asymptote at  $y = -1$

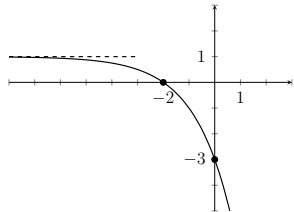
10. If  $\log_3(x) + \log_3(x - 1) = 1 + 3\log_3(x + 1)$ , then

- A.  $\frac{x^2-x}{(x+1)^3} = 3$   
 B.  $\frac{x}{(x-1)(x+1)} = 1$   
 C.  $x(x - 1) = 1 + (x + 1)^3$   
 D.  $x(x - 1)(x + 1)^3 = 3$

11. If 1200 dollars are invested at 1.75% interest compounded biweekly, how much is in the account after 5 years?

- A.  $1200 \cdot \left(1 + \frac{1.75}{26}\right)^{26 \cdot 5}$   
 B.  $1200 \cdot \left(1 + \frac{0.0175}{52}\right)^{52 \cdot 5}$   
 C.  $1200 \cdot \left(1 + \frac{0.0175}{26}\right)^{26 \cdot 5}$   
 D.  $1200 \cdot \left(1 + \frac{1.75}{104}\right)^{104 \cdot 5}$

12. Find an equation for the following graph.



- A.  $y = -2^{x+2} + 1$   
 B.  $y = 1 + 2^{x+2}$   
 C.  $y = 2^{x+2} - 1$   
 D.  $y = 1 - 2^{2-x}$

## Short Answer

15. Solve this equation for  $x$ :  $2(x + 2) - 5(x - 1) = 6(x - 3) + 9(x - 5)$

16. Simplify as much as possible:  $\frac{(4x^{-1}y)^2}{2x^2y}$ .

17. Solve for  $x$ :  $x^{-1/2} = 3$ .

18. Simplify  $\frac{\frac{1}{x+2} + 3}{\frac{x}{x+2} + 3}$ .

13. Suppose  $\theta$  is an angle in standard position such that  $\tan \theta < 0$  and  $\cos \theta < 0$ . In which quadrant will we find the terminal side of  $\theta$ ?

- A. I  
 B. II  
 C. III  
 D. IV

14. Suppose  $\tan \theta$  is undefined. Which of the following will also be undefined?

- A.  $\cos \theta$   
 B.  $\cot \theta$   
 C.  $\csc \theta$   
 D.  $\sec \theta$

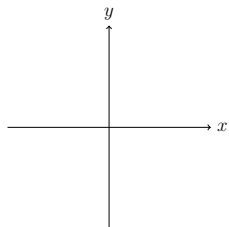
19. If  $\log_4(x) = C \log_3(x)$ , then  $C = \frac{\ln A}{\ln B}$  with  $A$  and  $B$  real numbers. Find  $A$  and  $B$ .

20. Change to exponential form:  $\ln(x) = 2$ .

21. Simplify:  $10^{3 \log(e) - \log(e^2) - \frac{1}{2} \log(e)}$ .

22. Consider the angle  $\theta = 5\pi/6$ .

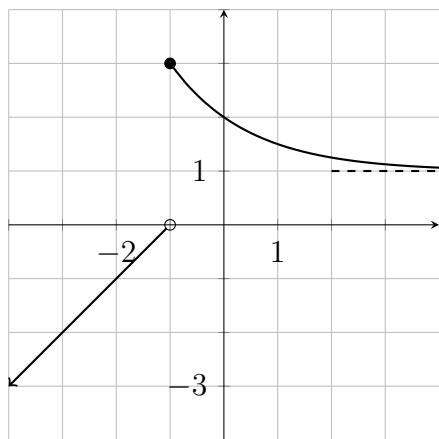
(a) Sketch  $\theta$  below in standard position.



(b) Find the exact value of  $\sec \theta$ .

## Long Answer

23. You are given the function  $f(x)$  below.



(a) What is the domain of  $f$ ?

(b) What is the range of  $f$ ?

(c) Find  $f(f(-2))$ .

(d) Is  $f$  invertible?

24. Find  $f^{-1}(x)$  if  $f(x) = \frac{2x+1}{3x-4}$ .

25. Factor completely:  $5x^3 - 10x^2 - 5x + 10$ .

26. (a) Find the  $x$  and  $y$ -intercepts of the function  $g(x) = x^2 - 8x + 9$ .

(b) Find the vertex of the graph of  $g$  by completing the square.

(c) Sketch a graph of the function  $g$ .

27. State the domain of  $(f/g)(x)$ , then simplify  $(f/g)(x)$  given that

$$f(x) = \frac{4x^2 - 4x - 15}{2x^2 - 5x} \quad \text{and} \quad g(x) = \frac{(2x+3)(x+3)}{(x+3)(x+1)}$$

28. Solve  $\frac{x+8}{x^3-9x} = \frac{2}{x^2+3x} - \frac{3}{x^2-9}$ .

29. Solve for  $x$  in each of the following equations:

(a)  $x^2 - 45x + 126 = 0$ ;

(b)  $\sqrt{7x-5} + \sqrt{3x-5} = 6$ ;

(c)  $\sqrt{7x-5} - \sqrt{3x-5} = 6$ .

30. Write the following expression in terms of logs of  $x$ ,  $y$  and  $z$ :

$$\log \sqrt{\frac{xy^2}{z^8}}.$$

31. Solve for  $x$ :  $3 + \log_5(10) + \log_5(x) = \log_5(3x-1) + 4$ .

32. Given  $f(x) = -\log_3(-x) + 1$ , answer the following.

(a) Identify any intercept(s) that occur.

(b) Write the equation of any asymptotes.

(c) Sketch  $y = f(x)$ .

(d) Is  $y = f(x)$  invertible? justify your answer. If  $f(x)$  is invertible, find the inverse function  $f^{-1}$ .

33. Solve the equation  $1 - \frac{5^{2x+1}}{3^{4-x}} = 0$ . Give your answer in form  $x = \frac{\ln A}{\ln B}$ .

34. Find all angles in  $[0, 2\pi)$  that satisfy the given equation:  $2 \cos^2 \theta - \cos \theta = 0$ .

35. Simplify:  $\cos x + \sin x \tan x$ .

36. Given  $y = -2 \cos(\pi x)$ , state the amplitude and period of this function, and sketch its graph. Include two cycles, and clearly label the axes.

## Applications

37. A zombie outbreak starts in Serbia with 3 cases in a remote village. After 5 days, a frightening total of 12 zombies has been reported. Let us assume that the number of zombies  $N(t)$  after  $t$  days grows exponentially ( $N(t) = k \cdot b^t$ ).

(a) Find the value of  $k$  using the fact that the initial 3 cases happen at  $t = 0$ .

(b) Find the value of  $b$ .

(c) How many zombies will there be after 10 days?

(d) After how many days will the entire population of 768 people in the village be converted into zombies if no cure is found?

38. A security camera in a neighborhood bank is mounted on a wall 9 feet above the floor. What angle of depression should be used if the camera is to be directed to a spot 6 feet above the floor and 12 feet from the wall?

## Answers

### Multiple choice

1. D      3. C      5. D      7. B      9. B      11. C      13. B  
 2. A      4. C      6. C      8. D      10. A      12. A      14. D

### Short Answer

15.  $x = 4$

19.  $C = \frac{\ln 3}{\ln 4}$

16.  $\frac{8y}{x^4}$

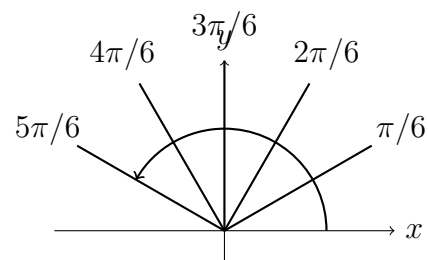
20.  $x = e^2$

17.  $x = 1/9$

18.  $\frac{3x + 7}{4x + 6}$

21.  $\sqrt{e}$

22. (a)



(b)  $\sec(5\pi/6) = -2/\sqrt{3}$

### Long Answer

23. (a) Domain =  $\mathbb{R}$

(b) Range =  $(-\infty, 0) \cup (1, 3]$

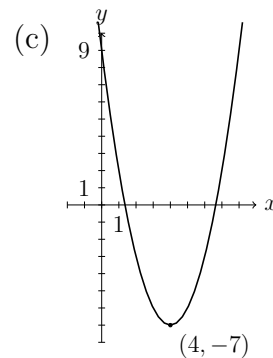
(c)  $f(f(-2)) = 3$

(d) yes (passes the horizontal line test)

24.  $f^{-1}(x) = \frac{4x + 1}{3x - 2}$

25.  $5(x - 1)(x + 1)(x - 2)$

26. (a)  $y$ -intercept:  $(0, 9)$ ,  $x$ -intercepts:  $(4 - \sqrt{7}, 0)$  and  $(4 + \sqrt{7}, 0)$

(b) vertex at  $(4, -7)$ 

27. The domain of  $(f/g)$  is  $\mathbb{R} \setminus \{0, \frac{5}{2}, \frac{-3}{2}, -3, -1\}$ ,  $(f/g)(x) = \frac{x+1}{x}$

28.  $x = -7$

29. (a)  $x = 3, 42$

(b)  $x = 3$

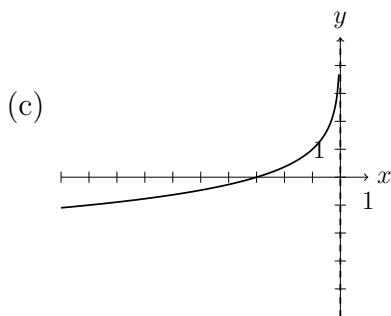
(c)  $x = 42$

30.  $\frac{1}{2} \log x + \log y - 4 \log z$

31.  $x = 1$

32. (a)  $x$ -intercept  $(-3, 0)$

(b) vertical asymptote  $x = 0$



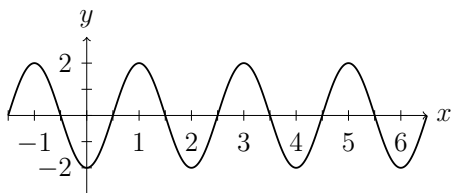
(d)  $f(x)$  is invertible (passes the horizontal line test),  $f^{-1}(x) = -3^{1-x}$ .

33.  $x = \frac{\ln(81/5)}{\ln(75)}$

34.  $\theta = \pi/3, \pi/2, 3\pi/2, 5\pi/3$

35.  $\sec x$

36.  $A = 2, P = 2$ .



## Applications

37. (a)  $k = 3$

(b)  $b = 4^{1/5}$ .

(c) 48

(d) 20

38.  $14.04^\circ$