1. (12 points) Evaluate the following limits.

(a) $\lim\limits_{x\to 2^-} \dfrac{2x^3-4x^2}{3x^2-8x+4}$

(b) $\lim\limits_{x\to 0} \dfrac{\sin^2(3x)}{5x\sin(2x)}$

(c) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{4x^6+3x^5}}{2x^3+\sqrt{9x^6+7x^5}}$

(d) $\lim\limits_{x\to 2^+} \dfrac{\sqrt{x-2}-(x-2)}{6-3x}$

2. (4 points) Let

$$f(x)=\begin{cases} x^2-k-3, & x<-1 \\ k+4, & x=-1 \\ k^2+4x-4, & x>-1 \end{cases}$$

(a) Find all values for k such that $\lim\limits_{x\to -1} f(x)$ exists.

(b) Find all values for k that make f continuous at all points.

3. (4 points) Let $f(x)=\dfrac{1}{3-2x}$. Use the limit definition of derivative to find $f'(x)$.

4. (16 points) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answer.

(a) $y=16\sqrt[4]{x}+e^x-x^e+\dfrac{\pi}{x}$

(b) $y=\dfrac{(8-5x^2)^4}{\tan(7x)-9}$

(c) $y=e^{\sqrt{2x^3}}$

(d) $y=(\sin x)^{4\ln x}$

5. (4 points) Write an equation of the tangent line to the curve

$$x^2y+\sin y+\frac{4}{\pi}y=3e^x$$

at the point $(0,\pi/2)$.

6. (6 points) Let θ (in radians) be an acute angle in a **right** triangle and let x and y be, respectively, the lengths of the sides adjacent and opposite to θ. Suppose also that x and y vary with time. At a certain instant, $x=4$ cm and increasing at 8 cm/s, while $y=3$ cm and is decreasing at 2 cm/s. How fast is θ changing at that instant?

7. (6 points) A box with a square base and open top needs to be made. The material for the base of the box costs \$10 per square meter, while the material for the sides cost \$5 per square meter. Using only \$120 what are the dimensions of such a box with largest volume?

8. (6 points) Find the absolute extrema of $f(x)=\dfrac{x}{2}+\dfrac{2}{x^2}$ on the interval $[1,4]$.

9. (6 points) The function $s(t) = t^3 - 3t^2$ describes the position of a particle moving along a coordinate line, where s is in meters and $t \geq 0$ is in seconds.

(a) Find the velocity function.

(b) At what times is the particle at rest?

(c) When is the particle moving in the positive direction?

10. (10 points) Consider the following function, along with its two first derivatives.

$$f(x) = \frac{x+2}{\sqrt{x^2+2}}, \quad f'(x) = \frac{2(1-x)}{(x^2+2)^{3/2}}, \quad f''(x) = \frac{2(x-2)(2x+1)}{(x^2+2)^{5/2}}.$$

(It might help to know that $f(-1/2) = 1$, $f(0) \approx 1.41$, $f(1) \approx 1.73$, and $f(2) \approx 1.63$.)

(a) Find the domain and intercepts of f.

(b) Find the vertical and horizontal asymptotes of f (if any).

(c) Find the intervals of increase/decrease of f.

(d) Find the local (relative) extrema of f.

(e) Find the intervals of concavity of f.

(f) Find all points of inflection of f.

(g) On the next page, sketch a graph of f.

11. (16 points) Evaluate each of the following integrals.

(a) $\displaystyle\int \left(\frac{2}{x} - \sqrt[3]{x^5} + 7e^x \right) \, dx$

(b) $\displaystyle\int \frac{(5x-3)^2}{x} \, dx$

(c) $\displaystyle\int \frac{1 - \sin\theta}{\cos^2\theta} \, d\theta$

(d) $\displaystyle\int_2^3 \frac{x^2 + 8x + 15}{x+3} \, dx$

12. (4 points) Given $f(x) = \displaystyle\int_6^{1/x} \frac{t}{\sqrt{1+t}} \, dt$, find:

(a) $f(1/6)$

(b) $f'(x)$

13. (4 points) Express $\displaystyle\int_0^5 \sin(x^2) \, dx$ as the limit of a Riemann sum. Do not evaluate the limit.

14. (2 points) Decide whether the equality below is correct or not. Justify.

$$\int \ln x \, dx = x \ln x - x + C$$

Answers:

1. (a) 2
(b) 9/10
(c) 2
(d) $-\infty$

2. (a) $2, -3$
(b) -3

3. $2/(3-2x)^2$

4. (a) $4x^{-3/4} + e^x - ex^{e-1} - \pi/x^2$
(b) $\frac{4(8-5x^2)^3(\tan(7x)-9)-(8-5x^2)^4(7\sec^2(7x))}{(\tan(7x)-9)^2}$
(c) $e^{\sqrt{2x^3}} 3x^2/\sqrt{2x^3}$
(d) $(\sin x)^{4\ln x}\left[\frac{4}{x}\ln(\sin x) + \frac{4\ln x\ cosx}{\sin x}\right]$

5. $y = \frac{3\pi}{4}x + \frac{\pi}{2}$

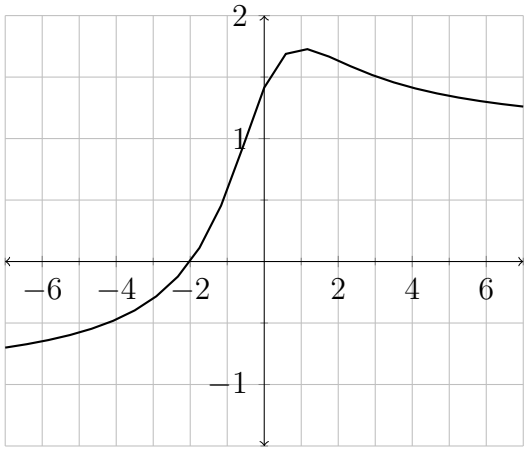
6. $-32/25\ rad/s$

7. $2 \times 2 \times 2$

8. abs. max. $= 2.5$ / abs. min. $= 1.5$

9. (a) $v(t) = 3t^2 - 6t$
(b) $t = 0$, $t = 2$
(c) when $t > 2$

10. (a) domain: $\mathbb{R}$; x-int.: $(-2, 0)$; y-int.: $(0, \sqrt{2})$
(b) h.a.: $y = -1$, $y = 1$; v.a.: none
(c) inc.: $(-\infty, 1)$; dec.: $(1, \infty)$
(d) local max. at $x = 1$; no local min.
(e) conc. up: $(-\infty, -1/2)$, $(2, \infty)$; conc. down: $(-1/2, 2)$
(f) inflection pts at $x = -1/2$, $x = 2$
(g)



11. (a) $2\ln|x| - \frac{3}{8}x^{8/3} + 7e^x + C$
(b) $\frac{25}{2}x^2 - 30x + 9\ln|x| + C$
(c) $\tan\theta - \sec\theta + C$
(d) 15/2

12. (a) 0
(b) $\frac{1/x}{\sqrt{1+1/x}}\frac{-1}{x^2}$

13. $\lim_{n\to\infty} \sum_{i=1}^{n} \sin(25i^2/n^2)5/n$

14. Correct. (Because $[x\ln x - x + C]' = \cdots = \ln x$.)