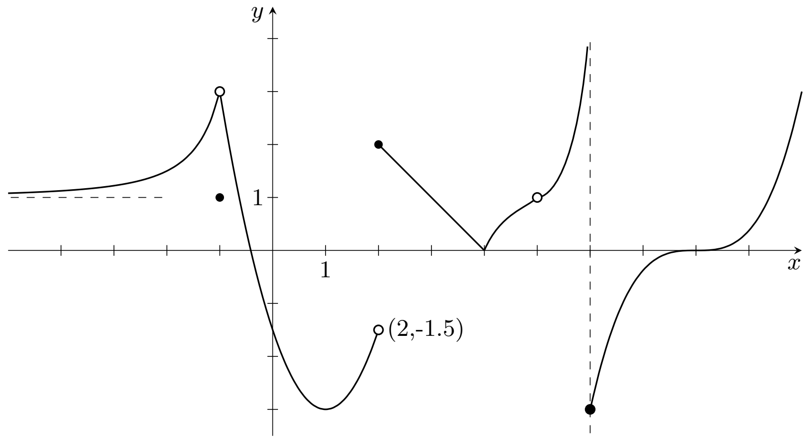
1. For the function f whose graph is shown below, determine each of the following. Use "does not exist" (*DNE*), ∞ or $-\infty$, where appropriate.



   (a) $\lim_{x\to -1} f(x) =$

   (b) $\lim_{x\to 2^-} f(x) =$

   (c) $\lim_{x\to 2} f(x) =$

   (d) $\lim_{x\to 6^-} f(x) =$

   (e) $\lim_{x\to 6^+} f(x) =$

   (f) $\lim_{x\to -\infty} f(x) =$

   (g) $\lim_{x\to \infty} f(x) =$

   (h) $\lim_{x\to 4} \dfrac{1}{f(x)} =$

   (i) $f(-1) =$

   (j) $f(6) =$

   (k) List all $x-$values where the function is discontinuous.

   (l) List all $x-$values where the function is continuous but not differentiable.

2. Evaluate the following limits. Use "does not exist" (*DNE*), ∞ or $-\infty$, where appropriate.

   (a) $\lim_{x\to 1} \dfrac{2x^2+x-3}{x^2+7x-8}$

   (b) $\lim_{x\to 1} \dfrac{\frac{1}{x+2}+\frac{1}{x-4}}{x-1}$

   (c) $\lim_{x\to -4} \dfrac{1-\sqrt{x+5}}{x+4}$

   (d) $\lim_{x\to 1^-} \dfrac{3(x^2-1)}{|1-x|}$

   (e) $\lim_{x\to \infty} \dfrac{(3-2x)(x^2-3x+2)}{5x(x^2+1)}$

   (f) $\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} x^2-3 & \text{if} \quad x \le 2 \\ 3x-1 & \text{if} \quad 2 < x \end{cases}$

3. Use the definition of continuity to determine the points of discontinuity of the following function:

$$f(x) = \begin{cases} \frac{x-2}{x^2+x-6} & \text{if} \quad x < 2 \\ \frac{1}{x+3} & \text{if} \quad 2 \le x \le 4 \\ \frac{1}{x-5} & \text{if} \quad 4 < x \end{cases}$$

4. Find the value(s) of k for which the following function is continuous on $\mathbb{R}$.
$f(x) = \begin{cases} x^2 + k^2 x & \text{if} \quad x \leq 1 \\ 5k + 7x & \text{if} \quad x > 1 \end{cases}$

5. **Use the limit definition** of the derivative to calculate the derivative of $f(x) = \dfrac{3}{2x-1}$

6. Compute $\frac{dy}{dx}$ for each of the following equations. Use properties of logarithms where appropriate. Do NOT simplify your answers.
   (a) $y = \pi x^4 - \frac{2}{x} + \sqrt[3]{x^2} + \log_5(\sin x) + \ln 2$
   (b) $y = \dfrac{8x^4}{1 + \tan x}$
   (c) $y = 2(7 - e^{2x^3})^5(5 - 6x^4)^3$
   (d) $(x - y)^2 + 4x - 5y - 1 = 0$
   (e) $y = \ln\left(\dfrac{3x^5 \cot^4 x}{(2x)^3(3x^4 + 5)^8}\right)$
   (f) $y = (x^3 - 4x)^{\sec(6x)}$

7. Suppose f and g are differentiable functions such that $f'(4) = -1, f'(5) = -3, g(2) = 5$, and $g'(2) = 4$. If $h(x) = f(g(x))$, compute $h'(2)$.

8. Given $2x^2 - 3xy + 3y^2 = 2$, find an equation of the line tangent to the curve at the point $(1, 1)$.

9. Compute the 81^{st} derivative of $f(x) = 8\sin(1 - x) - 5x^{74}$.

10. Find the absolute extrema of $f(x) = \sqrt[3]{x}(x^2 - 7)$ on the interval $[-8, 0]$.

11. Use the second derivative test to find the local extrema of $f(x) = 2x^5 - 30x^3 + 7$. If the test fails, simply state this.

12. Consider $f(x) = \dfrac{2x^2}{x^2 - 1}$, with $f'(x) = \dfrac{-4x}{(x^2 - 1)^2}$, and $f''(x) = \dfrac{4(3x^2 + 1)}{(x^2 - 1)^3}$.
Determine the following, then neatly sketch the graph of $f(x)$ on the following page. Clearly label any important points.
   (a) the domain of f,
   (b) all vertical and horizontal asymptotes,
   (c) all $x-$ and $y-$intercepts,
   (d) the intervals on which f is increasing and decreasing,
   (e) all local extrema of f,
   (f) the intervals on which f is concave up and concave down,
   (g) the inflection points of f,
   (h) sketch the graph.

13. Suppose the **average cost** (in dollars) to produce x tea pots is given by the function $\overline{C}(x) = x^2 - 4x + \frac{10}{x} + 8$. Compute the **marginal cost** when $x = 10$ and **interpret the result**.

14. A company wants to enclose a storage area with a fence, next to a wall of a building. The storage area will be 6400 square meters. The fence opposite the wall of the building costs \$10 per meter and the fence on the other two sides costs \$20 per meter. Find the dimensions of the storage area to minimize the cost of the fence.

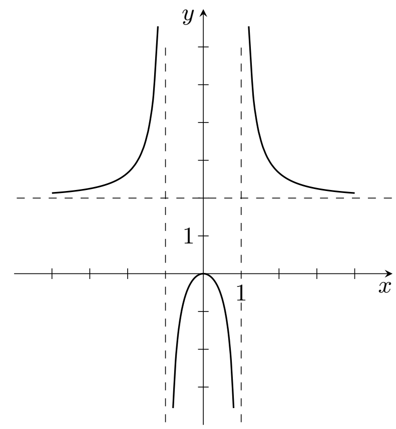
15. A company owns an apartment building containing 100 units. If the monthly rent the company charges for each unit is \$400, then all units can be rented out. For every \$20 increase in rent, the company will lose one customer. What rent should be charged per unit to maximize revenue?

16. Suppose the demand equation for potatoes is given by the equation $100 - x = p^2$ where x is measured in kg of potatoes.
    (a) Compute the elasticity of demand function, $E(p)$.
    (b) If the current price is \$5 per kg of potatoes, what effect will a 6% increase in price have on the demand for potatoes?
    (c) What price per kg of potatoes will maximize the revenue obtained from potato sales? Round your answer to the nearest cent.

ANSWERS

1. (a)3, (b) -1.5, (c) DNE, (d) ∞, (e) -3, (f) 1, (g) ∞, (h) ∞, (i) 1, (j) -3, (k) -1, 2, 5, 6, (l) 4
2. (a) $\frac{5}{9}$, (b) $-\frac{2}{9}$, (c) $-\frac{1}{2}$, (d) -6, (e) $-\frac{2}{5}$, (f) DNE
3. Discontinuity at -3, 4 and 5
4. $k = -1$ or $k = 6$
5. $f'(x) = -\frac{6}{(2x-1)^2}$
6. (a) $\frac{dy}{dx} = 4\pi x^3 + \frac{2}{x^2} + \frac{2}{3\sqrt[3]{x}} + \frac{\cos x}{(\ln 5)\sin x}$
    (b) $\frac{dy}{dx} = \frac{32x^3(1+\tan x)-8x^4\sec^2 x}{(1+\tan x)^2}$
    (c) $\frac{dy}{dx} = -60(7-e^{2x^3})^4x^2e^{2x^3}(5-6x^4)^3 - 144(7-e^{2x^3})^5(5-6x^4)^2x^3$
    (d) $\frac{dy}{dx} = \frac{2y-2x-4}{-2x+2y-5}$
    (e) $\frac{dy}{dx} = \frac{2}{x} - \frac{4\csc^2 x}{\cot x} - \frac{96x^3}{3x^4+5}$
    (f) $\frac{dy}{dx} = (x^3-4x)^{\sec(6x)}\left[6\sec(6x)\tan(6x)\ln(x^3-4x) + \frac{(3x^2-4)\sec(6x)}{x^3-4x}\right]$
7. -12
8. $y = -\frac{1}{3}x + \frac{4}{3}$
9. $f^{(81)}(x) = -8\cos(1-x)$.
10. absolute max. $= 6$ at $x = -1$, absolute min. $=-114$ at $x = -8$
11. local max. $= 331$ at $x = -3$, local min. $= -317$ at $x = 3$, test fails at $x = 0$

12.



(a) $x \neq \pm 1$,

(b) V.A. $x = \pm 1$, H.A. $y = 2$,

(c) $x-$ and $y-$intercept $(0, 0)$,

(d) increasing on $(-\infty, -1), (-1, 0)$ decreasing on $(0, 1), (1, \infty)$,

(e) local max. at $(0, 0)$ no local min.,

(f) concave up on $(-\infty, -1)$ and $(1, \infty)$, concave down on $(-1, 1)$,

(g) no inflection points

(h)

13. Marginal cost $C'(10) = 228\$$ is the cost of producing 11th tea pot.

14. 160 m (opposite the wall) by 40m

15. \$1200,

16. (a) $E(p) = \frac{2p^2}{100-p^2}$, (b) the demand will decrease by 4 %, (c) $p \approx \$5.77$.