Multiple choice

For each of the following questions, there is only one correct answer. Circle your choice. If two choices are selected for the same question, no marks will be awarded.

1. The line $y = 2x + 1$ has the same slope but a different y-intercept as which one of these lines?

A. $2y = x$

B. $x = \frac{y-1}{2}$

C. $y = 2$

D. $x = \frac{y-4}{2}$

2. Which of these functions has a domain of $(1, \infty)$?

A. $y = \frac{1}{\sqrt{x-1}}$

B. $y = \frac{1}{(x-1)^2}$

C. $y = \sqrt{x-1}$

D. $y = \frac{1}{x-1}$

3. Suppose $ax^2+bx+c = 0$ has exactly one solution. Then:

A. $b^2 - 4ac > 0$

B. $b^2 - 4ac < 0$

C. $b^2 - 4ac = 0$

D. None of the above.

4. Given that $f(g(x)) = (x + 3)^2 - 1$, then f and g can be:

A. $f(x) = x + 3, g(x) = x^2 - 1$

B. $f(x) = x^2 + 3, g(x) = x - 1$

C. $f(x) = x^2 - 1, g(x) = x + 3$

D. $f(x) = (x + 3)^2, g(x) = x - 1$

5. Assuming all variables are positive then $\sqrt{72x^3y^6}$ equals

A. $2x^2y^3\sqrt{6y}$

B. $12xy^3\sqrt{x}$

C. $6xy^3\sqrt{2x}$

D. $12xy^2\sqrt{xy^3}$

E. $6xy\sqrt{2xy^3}$

6. True or False $f(x) = \dfrac{(x-3)(x+2)}{(x+1)(x-3)}$ and $g(x) = \dfrac{x+2}{x+1}$ have the same domain.

A. T

B. F

7. To rationalize the denominator of $f(x) = \dfrac{3x+7}{\sqrt{4x}-1}$ to produce an expression equal to $f(x)$, you should

A. multiply the denominator of $f(x)$ by $\sqrt{4x}$.

B. multiply both the numerator and the denominator of $f(x)$ by $\sqrt{4x}$.

C. multiply the denominator of $f(x)$ by $\sqrt{4x}+1$.

D. multiply both the numerator and the denominator of $f(x)$ by $\sqrt{4x}+1$.

E. multiply the denominator of $f(x)$ by $\sqrt{4x}-1$.

F. multiply both the numerator and the denominator of $f(x)$ by $\sqrt{4x}-1$.

8. The domain of $f(x) = \sqrt{\dfrac{(x-1)(x+3)}{x(x+1)}} + 3x + 1$ is

A. $[-3,-1) \cup (0,1]$

B. $[-3,-1) \cup (0,1) \cup (1,+\infty)$

C. $(-\infty,-3) \cup (0,1) \cup (1,+\infty)$

D. $(-\infty,-3] \cup (-1,0) \cup [1,+\infty)$

9. (1 point) The graph of $y = \log(x + 1) - 2$ has

A. a vertical asymptote at $x = -2$

B. a vertical asymptote at $x = -1$

C. a horizontal asymptote at $y = -2$

D. a horizontal asymptote at $y = -1$

10. (1 point) If $\log_3(x) + \log_3(x-1) = 1 + 3\log_3(x+1)$, then

A. $\frac{x^2-x}{(x+1)^3} = 3$

B. $\frac{x}{(x-1)(x+1)} = 1$

C. $x(x-1) = 1 + (x+1)^3$

D. $x(x-1)(x+1)^3 = 3$

11. If 1200 dollars are invested at 1.75% interest compounded biweekly, how much is in the account after 5 years?

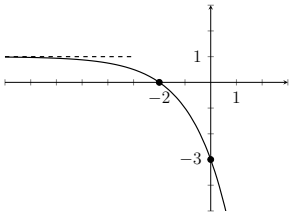
A. $1200 \cdot \left(1 + \dfrac{1.75}{26}\right)^{26\cdot 5}$

B. $1200 \cdot \left(1 + \dfrac{0.0175}{52}\right)^{52\cdot 5}$

C. $1200 \cdot \left(1 + \dfrac{0.0175}{26}\right)^{26\cdot 5}$

D. $1200 \cdot \left(1 + \dfrac{1.75}{104}\right)^{104\cdot 5}$

12. Find an equation for the following graph.



A. $y = -2^{x+2} + 1$

B. $y = 1 + 2^{x+2}$

C. $y = 2^{x+2} - 1$

D. $y = 1 - 2^{2-x}$

13. Suppose θ is an angle in standard position such that $\tan\theta < 0$ and $\cos\theta < 0$. In which quadrant will we find the terminal side of θ?

A. I

B. II

C. III

D. IV

14. Suppose $\tan\theta$ is undefined. Which of the following will also be undefined?

A. $\cos\theta$

B. $\cot\theta$

C. $\csc\theta$

D. $\sec\theta$

Short Answer

1. Solve this equation for x.

$$2(x+2) - 5(x-1) = 6(x-3) + 9(x-5)$$

2. Simplify as much as possible: $\dfrac{(4x^{-1}y)^2}{2x^2y}$.

3. If $x^{-1/2} = 3$ then solve for x.

4. Simplify $\dfrac{\frac{1}{x+2} + 3}{\frac{x}{x+2} + 3}$.

5. Multiply and simplify $\left(2\sqrt{x+1}+3\sqrt{x}\right)\left(2\sqrt{x+1}-3\sqrt{x}\right)$.

6. Reduce and simplify $5\sqrt{18x}-4\sqrt{8x}+3\sqrt{50x}$.

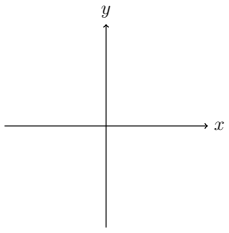
7. If $\log_4(x) = C\log_3(x)$, then $C = \dfrac{\ln A}{\ln B}$ with A and B real numbers. Find A and B.

8. Change to exponential form: $\ln(x) = 2$.

9. Simplify: $10^{3\log(e)-\log(e^2)-\frac{1}{2}\log(e)}$.

10. Consider the angle $\theta = 5\pi/6$.

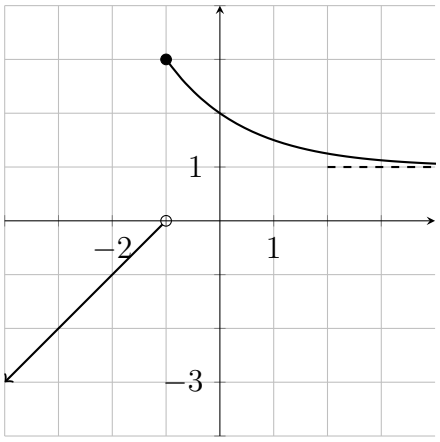
(a) Sketch θ below in standard position.



(b) Find the exact value of $\sec\theta$.

Long Answer

1. You are given the function $f(x)$ below.



(a) What is the domain of f?

(b) What is the range of f?

(c) Find $f(f(-2))$.

(d) Is f invertible?

2. Find $f^{-1}(x)$ if $f(x) = \dfrac{2x+1}{3x-4}$.

3. Factor completely: $5x^3 - 10x^2 - 5x + 10$.

4. (a) Find the x and y-intercepts of the function $g(x) = x^2 - 8x + 9$.

(b) Find the vertex of the graph of g by completing the square.

(c) Sketch a graph of the function g.

5. State the domain of $(f/g)(x)$, then simplify $(f/g)(x)$ given that $f(x) = \dfrac{4x^2 - 4x - 15}{2x^2 - 5x}$ and $g(x) = \dfrac{(2x+3)(x+3)}{(x+3)(x+1)}$

6. Solve $\dfrac{x+8}{x^3 - 9x} = \dfrac{2}{x^2 + 3x} - \dfrac{3}{x^2 - 9}$.

7. Solve for x in each of the following equations:

(a) $x^2 - 45x + 126 = 0$;

(b) $\sqrt{7x-5} + \sqrt{3x-5} = 6$;

(c) $\sqrt{7x-5} - \sqrt{3x-5} = 6$.

8. Write the following expression in terms of logs of x, y and z:

$$\log \sqrt{\frac{xy^2}{z^8}}.$$

9. Solve for x, if possible:

$$3 + \log_5(10) + \log_5(x) = \log_5(3x - 1) + 4.$$

10. Given $f(x) = -\log_3(-x) + 1$, answer the following.

(a) Identify any intercept(s) that occur.

(b) Write the equation of any asymptotes.

(c) Sketch y = $f(x)$.

(d) Is $y = f(x)$ invertible? justify your answer. If $f(x)$ is invertible, find the inverse function f^{-1}.

11. Solve the equation $1 - \dfrac{5^{2x+1}}{3^{4-x}} = 0$. Give your answer in form $x = \dfrac{\ln A}{\ln B}$.

12. Find all angles in $[0, 2\pi)$ that satisfy the given equation: $2\cos^2\theta - \cos\theta = 0$.

13. Given $y = -2\cos(\pi x)$, state the amplitude and period of this function, and sketch its graph. Include two cycles, and clearly label the axes.

Applications

1. A zombie outbreak starts in Serbia with 3 cases in a remote village. After 5 days, a frightening total of 12 zombies has been reported. Let us assume that the number of zombies $N(t)$ after t days grows exponentially ($N(t) = k \cdot b^t$).

(a) Find the value of k using the fact that the initial 3 cases happen at $t = 0$.

(b) Find the value of b.

(c) How many zombies will there be after 10 days?

(d) After how many days will the entire population of 768 people in the village be converted into zombies if no cure is found?

2. A security camera in a neighborhood bank is mounted on a wall 9 feet above the floor. What angle of depression should be used if the camera is to be directed to a spot 6 feet above the floor and 12 feet from the wall?

Answers

Multiple choice

1. D	**3.** C	**5.** C	**7.** D	**9.** B	**11.** C	**13.** B
2. A	**4.** C	**6.** B	**8.** D	**10.** A	**12.** A	**14.** D

Short Answer

1. $x = 4$

2. $\dfrac{8y}{x^4}$

3. $x = 1/9$

4. $\dfrac{3x+7}{4x+6}$

5. $4 - 5x$

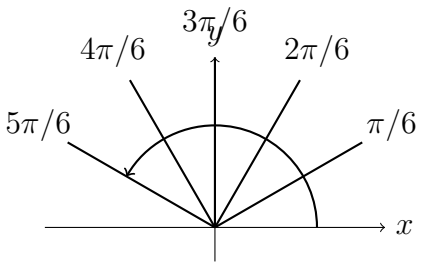
6. $24\sqrt{2x}$

7. $C = \dfrac{\ln 3}{\ln 4}$

8. $x = e^2$

9. $\sqrt{e}$

10. (a)



(b) $\sec(5\pi/6) = -2/\sqrt{3}$

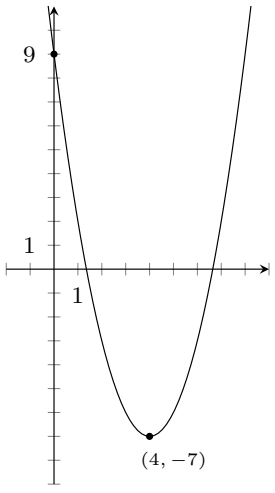
Long Answer

1. (a) Domain $= \mathbb{R}$
(b) Range $= (-\infty, 0) \cup (1, 3]$
(c) $f(f(-2)) = 3$
(d) yes (passes the horizontal line test)

2. $f^{-1}(x) = \dfrac{4x+1}{3x-2}$

3. $5(x-1)(x+1)(x-2)$

4. (a) y-intercept: $(0, 9)$, x-intercepts: $(4 - \sqrt{7}, 0)$ and $(4 + \sqrt{7}, 0)$
(b) vertex at $(4, -7)$
(c)



5. The domain of (f/g) is $\mathbb{R} \setminus \{0, 5/2, -3/2, -3, -1\}$, $(f/g)(x) = \dfrac{x+1}{x}$

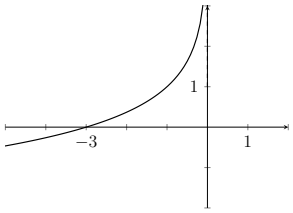
6. $x = -7$

7. (a) $x = 3, 42$
(b) $x = 3$
(c) $x = 42$

8. $\dfrac{1}{2}\log x + \log y - 4\log z$

9. $x = 1$

10. (a) x-intercept $(-3, 0)$
(b) vertical asymptote $x = 0$
(c)

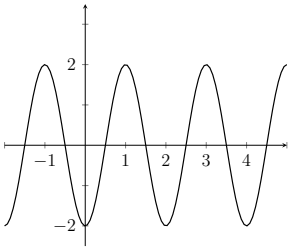


(d) $f(x)$ is invertible (passes the horizontal line test), $f^{-1}(x) = -3^{1-x}$.

11. $x = \dfrac{\ln(81/5)}{\ln(75)}$

12. $\theta = \pi/3, \pi/2, 3\pi/2, 5\pi/3$

13.



Applications

1. (a) $k = 3$
(b) $b = 4^{1/5}$.
(c) 48
(d) 20

2. 14.04°