1. (a) $f(x) = 4 + \dfrac{x^2}{3} + \displaystyle\sum_{n=2}^{\infty} \frac{8(-1)^{n-1}(1)(4)(7)\cdots(3n-5)x^{2n}}{(24)^n n!}$ $\quad R = 2\sqrt{2}$

   (b) $g(x) = \dfrac{\sin(2x)}{2} = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} x^{2n+1}}{(2n+1)!}$ $\quad R = \infty$

2. (a) $g(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n^2}$ $\quad R = 1$

   (b)

$$\begin{aligned} g(0.2) &= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0.2)^n}{n^2} \\ &\simeq 0.2 - \frac{(0.2)^2}{4} + \frac{(0.2)^3}{9} \\ &\simeq 0.19089 \end{aligned}$$

   $|error| \leq (0.2)^4/16 = 0.0001$

   (c) $g^{(7)}(0) = \dfrac{6!}{7}$

3. (a) $T_3(x) = 1 + 3(x-1) + \dfrac{7(x-1)^2}{2} + \dfrac{13(x-1)^3}{6}$

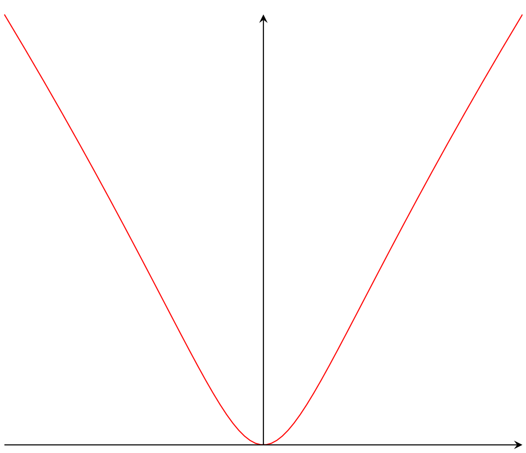
   $R_3(x) = \dfrac{(12 + 8z + z^2)e^{z-1}(x-1)^4}{4!}$ $\quad$ where z is between x and 1

   (b) $T_3(0.5) = \frac{5}{48} \simeq 0.104167$

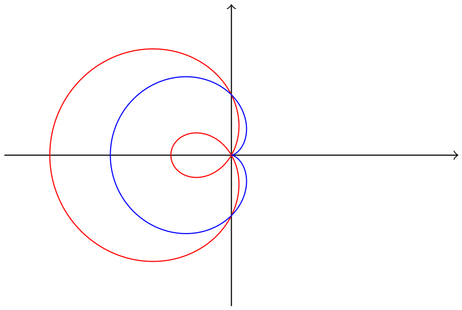
   (c) $|R_3(0.5)| \leq 0.0546$

4. (a) $\dfrac{dy}{dx} = \dfrac{-2t}{3(t^2+1)}$ and $\dfrac{d^2y}{dx^2} = \dfrac{2(1-t^2)}{9(t^2+1)^3}$

   (b) No vertical tangents; horizontal tangent at $(0,0)$ $(t = 0)$; (0,0) is the only intercept.

   (c)  The orientation is from right to left

   (d) $A = -\displaystyle\int_0^2 x\,dy = 2\int_0^2 (t^4 + 3t^2)dt$

5. (a) 

   (b) Points of intersection are $(0,0),(1,\pi/2),(1,3\pi/2)$

   (c) $A = \int_{\pi/2}^{\pi} \{(1-2\cos\theta)^2 - (1-\cos\theta)^2\} d\theta$

   (d)

$$\begin{aligned} \mathcal{L} &= \int_0^{2\pi} \sqrt{2-2\cos\theta} d\theta \\ &= \int_0^{2\pi} 2\sin(\theta/2) d\theta \\ &= 8 \end{aligned}$$

6. (a) $x = 1+t,\ y = \sqrt{2}\,t,\ z = 1-t$

   (b) $x + \sqrt{2}\,y - z = 0$

   (c) $\mathcal{L} = e - \dfrac{1}{e}$

   (d) $\kappa = \dfrac{\sqrt{2}}{(e^t + e^{-t})^2}$

   (e) $a_T = \dfrac{e^{2t} - e^{-2t}}{e^t + e^{-t}}$ and $a_N = \sqrt{2}$

7. (a) Parabolic Cylinder

   (b) Hyperboloid of two sheets opening on the y-axis

   (c) Sphere with center $(0,0,1/2)$ and radius $1/2$

8. (a) The limit does not exist

   (b) The limit is zero

9. (a) $\dfrac{\langle 3,4,2\rangle}{\sqrt{29}}$

   (b) $\sqrt{29}$

   (c) $3x + 4y + 2z = 9$

   (d) $\dfrac{\partial z}{\partial y} = -\dfrac{2z}{y}$

   (e) $D_{\mathbf{u}} f(1,1,1) = \frac{18}{\sqrt{21}}$

   (f) $\mathbf{r}'(1) \cdot \nabla f(1,1,1) = \langle -2,0,3\rangle \cdot \langle 3,4,2\rangle = 0$

10. (a) $df = \dfrac{yz}{2\sqrt{xyz}} dx + \dfrac{xz}{2\sqrt{xyz}} dy + \dfrac{xy}{2\sqrt{xyz}} dz$

    (b) $f(1.9, 2.02, 4.05) \simeq 4 - 0.055 = 3.945$

11. $x = 2 + 17t$, $y = 2 - 18t$, $z = 8 - 4t$

12. $\dfrac{\partial^2 z}{\partial x^2} = 2\dfrac{\partial f}{\partial u} + 4x^2\dfrac{\partial^2 f}{\partial u^2} + 4x\dfrac{\partial^2 f}{\partial v \partial u} + \dfrac{\partial^2 f}{\partial v^2}$

13. There are 4 critical points. The points $(0, 0)$, $(0, -1)$ and $(1, 0)$ are all saddle points and $(1/3, -1/3)$ is a local minimum.

14. The maximum is $f(1, -2, 5) = 30$ and the minimum is $f(-1, 2, -5) = -30$

15. (a) $\dfrac{1 - \cos(1)}{12}$

    (b) $\dfrac{4\pi}{3}$

16. $\displaystyle\int_0^1 \int_0^{1-z} \int_0^{y^2} dxdydz$

17. (a) $\displaystyle\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} \int_0^{x^2+y^2} dzdydx$

    (b) $\displaystyle\int_0^{\pi} \int_0^{2\cos\theta} \int_0^{r^2} rdzdrd\theta$

18. $\displaystyle V = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{2a\cos\phi} \rho^2 \sin\phi d\rho d\phi d\theta = \pi a^3$