Question 1: (*9 pts*) Evaluate the following limits:

a) $\lim\limits_{x\to\pi}\left(\dfrac{2x-\pi}{\pi}\right)^{2\csc(x)}$ **b)** $\lim\limits_{x\to 0^+}\dfrac{\arctan(\sqrt{x})}{4\sqrt{x}}$ **c)** $\lim\limits_{x\to+\infty} x\left(e^{3/x}-1\right)$

Question 2: (*9 pts*) Find the derivative of the following functions. **Do not simplify your answer.**

a) $y = 3\left(4-\arccos(3x)\right)^3$ **b)** $y=\arctan\left(\dfrac{1-x}{1+x}\right)$ **c)** $f(x)=\displaystyle\int_{e^x}^{4}\frac{\tan(t^2)}{\operatorname{arcsec}(3t+2)}\,dt$

Question 3: (*4 pts*) The altitude h (in m) of a rocket is given by $h(t) = -2t^3+84t^2+480t+10$, where t represents the time of flight (in seconds). Knowing that the rocket will hit the ground between 35s and 50s, use Newton's method to find when the rocket will hit the ground. Give an answer that is accurate to 3 decimals. (Show all your work!)

Question 4: (*5 pts*) Use Simpson's Rule to approximate $\displaystyle\int_3^4\sqrt{\ln(x)+3}\,dx$, using $n=6$. Give an answer that is accurate to 4 decimals.

Question 5: (*4 pts*) Computer simulations show that the drag F (in N) on a certain airplane is given by $F = 5.00\times10^{-3}v^2+\dfrac{3.00\times10^8}{v^2}$, where v is the velocity (in km/h) of the airplane. Find the velocity that will result in the minimum drag.

Question 6: (*4 pts*) An approximate relationship between the pressure P and volume V of the vapor in a diesel engine cylinder is given by $P\cdot V^{1.4}=k$, where k is a constant. At a certain instant, $P=4200$kPa, $V=75\text{cm}^3$, and the volume is increasing a the rate of $850\text{cm}^3/\text{s}$. At what rate is the pressure changing at this instant?

Question 7: (*12 pts*)

Given $f(x)=\dfrac{(x-1)^2(2x+1)}{x^3}$, $f'(x)=\dfrac{3(x^2-1)}{x^4}$ and $f''(x)=\dfrac{6(2-x^2)}{x^5}$, find (if any):

a) The domain of f.

b) The x and y intercept(s).

c) The vertical and horizontal asymptotes.

d) The intervals on which f is increasing or decreasing.

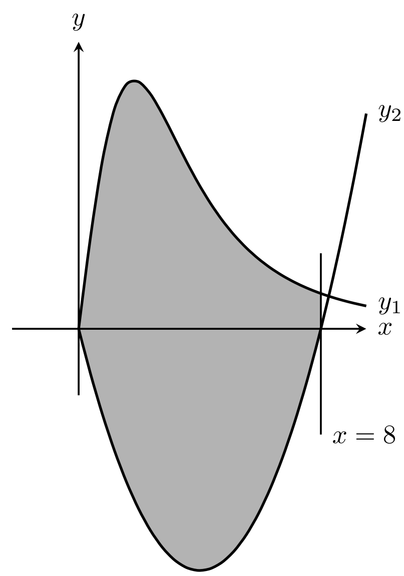
e) The local minima and maxima.

f) The intervals of upward and downward concavity.

g) The points of inflection.

h) Sketch the graph of f.

Question 8: (*4 pts*) Using CAD (Computer Assisted Design), an architect programs a computer to sketch the shape of a swimming pool, designed between the curves of $y_1 = \dfrac{800x}{(x^2+10)^2}$, $y_2 = 0.5x^2 - 4x$ and $x = 8$ (see figure). Find the surface area of the pool (the dimensions are in m).



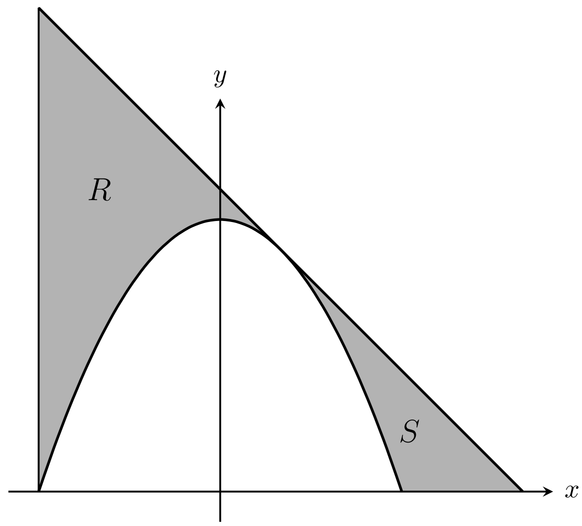
Question 9: (*28 pts*) Evaluate the following integrals:

a) $\displaystyle\int \frac{4x^3 - 15x^2 - 6x + 23}{x^2 - 3x - 4}\,dx$ **b)** $\displaystyle\int \tan^3(2x)\ \sec^5(2x)\ dx$ **c)** $\displaystyle\int_1^4 \frac{3x^2}{(x^3+80)^{3/2}}\,dx$

d) $\displaystyle\int_{-3}^{1} \frac{3x}{\sqrt{-x^2 - 6x + 55}}\,dx$ **e)** $\displaystyle\int \cos(x)\ln\Big(\sin(x)\Big)\,dx$ **f)** $\displaystyle\int (4x^2 - 2)\cos(2x)dx$

g) $\displaystyle\int \cot^6(x)\ \sin^9(x)\ dx$

Question 10: (*4 pts*) Consider the 2 regions R and S shown below. The parabola $y = 9 - \dfrac{x^2}{4}$ and the line $y = -x + 10$ (which is tangent to the parabola) border these 2 regions.



Set up, but **do not evaluate**, the integral to find the volume of the solid obtained by revolving:
a) the region S about the y-axis. **b)** the region R about the line $y = -2$.

Question 11: (*3 pts*) Determine whether or not the function $y = x^3 \ln(x)$ is a solution of the differential equation $x^2 y'' - xy' - 3y = 4x^3$.

Question 12: (*4 pts*) Solve the following separable differential equation: $\dfrac{dy}{dx} = xy\sqrt{1+x^2}$ with the initial condition $y(0) = -1$.

Question 13: (*4 pts*) Solve the following first order linear differential equation:

$$y' + 4y = 8x \quad \text{with initial condition} \quad y(0) = 0$$

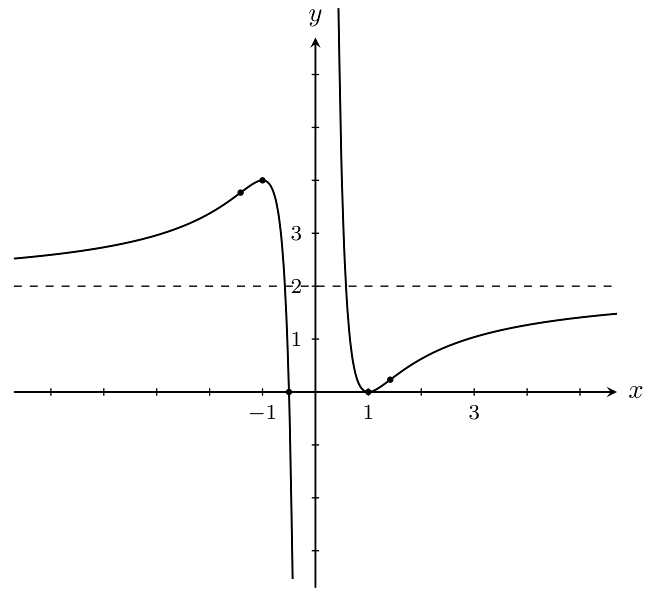
Question 14: (*6 pts*) Given the function $f(x) = \begin{cases} 0 & \text{if} \quad -\pi \leqslant x < 0 \\ 2 & \text{if} \quad 0 \leqslant x < \pi \end{cases}$ find a_0, a_2 and b_5 of the Fourier series of the function.

ANSWERS:

1.) **a)** $e^{-4/\pi}$ **b)** $\dfrac{1}{4}$ **c)** 3 **2.)** **a)** $\dfrac{27\left(4-\arccos(3x)\right)^2}{\sqrt{1-9x^2}}$ **b)** $\dfrac{-1}{x^2+1}$

c) $\dfrac{-e^x\tan(e^{2x})}{\operatorname{arcsec}(3e^x+2)}$ **3.)** $t=47.098\text{s}$ **4.)** 2.0613 **5.)** $V=494.92\text{km/h}$ **6.)** $-66\,640\text{kPa/s}$

7.) **a)** $\mathbb{R}\backslash\{0\}$ **b)** x-int: $(1,0)$ and $\left(-\frac{1}{2},0\right)$
y-int: none **c)** V.A.: $x=0$ H.A.: $y=2$
d) Inc. on $]-\infty,-1[$ and $]1,+\infty[$
Dec. on $]-1,0[$ and $]0.1[$ **e)** Local min.: $(1,0)$
Local max.: $(-1,4)$
f) C.U.: $]-\infty,-\sqrt{2}[$ and $]0,\sqrt{2}[$
C.D.: $]-\sqrt{2},0[$ and $]\sqrt{2},+\infty[$
g) $(-\sqrt{2},3.7678)$ and $(\sqrt{2},0.2323)$



8.) 77.26m^2 **9.)** **a)** $2x^2-3x+3\ln|x-4|-2\ln|x+1|+C$ **b)** $\dfrac{1}{14}\sec^7(2x)-\dfrac{1}{10}\sec^5(2x)+C$

c) $\dfrac{1}{18}$ **d)** $24-12\sqrt{3}-\dfrac{3\pi}{2}$ **e)** $\sin(x)\ln\Big(\sin(x)\Big)-\sin(x)+C$ **f)** $(2x^2-2)\sin(2x)+2x\cos(2x)+C$

g) $\dfrac{1}{9}\cos^9(x)-\dfrac{1}{7}\cos^7(x)+C$ **10.)** **a)** $\pi\displaystyle\int_0^8 (-y+10)^2-(36-4y)\;dy$

b) $\pi\displaystyle\int_{-6}^{2}(-x+12)^2-\left(11-\frac{x^2}{4}\right)^2\,dx$ **11.)** It is a solution. **12.)** $y=-e^{\left(\frac{1}{3}(1+x^2)^{3/2}-\frac{1}{3}\right)}$

13.) $y=2x-\dfrac{1}{2}+\dfrac{1}{2}e^{-4x}$ **14.)** $a_0=1$ $a_2=0$ $b_5=\dfrac{4}{5\pi}$