Question 1: (*27 pts*) Evaluate the following integrals.

a) $\displaystyle\int \frac{2t^2 - 4\sqrt{t} + t^{3/2}\csc(2t) - 5}{\sqrt{t^3}}\,dt$ **b)** $\displaystyle\int \frac{2\ln(x)}{x^3}\,dx$ **c)** $\displaystyle\int \frac{7\sec^2(4x)}{\sqrt{8 - 4\tan(4x)}}\,dx$ **d)** $\displaystyle\int_0^1 (x^2 - 2)e^{2x}\,dx$

e) $\displaystyle\int \frac{2\sec(x) - \cos(x) + 2}{\cot(x)}\,dx$ **f)** $\displaystyle\int_1^2 7 + |5 - 3x|\,dx$ **g)** $\displaystyle\int \frac{3x^4 - 4x^3 - 11x^2 + 17x - 12}{x^3 - x^2 - 6x}\,dx$

Question 2: (*4 pts*) Find $f(x)$, given that $f''(x) = 20x^3 + 12x^2 + 4$, and that the slope of the tangent line to the graph of f at the point $(1, 5)$ is 21.

Question 3: (*4 pts*) Use Riemann Sums with right end-points and $n = 4$ rectangles to approximate $\displaystyle\int_2^4 e^{\cos(2x)}\,dx$. Your answer should be accurate to 4 decimal places.

Remember: Your calculator should be in radians.

Question 4: (*3 pts*) Given that $\displaystyle\int_0^2 2f(x)\,dx = 6$ and $\displaystyle\int_7^2 f(x)\,dx = 4$, find $\displaystyle\int_0^7 6f(x) + 2x\,dx$.

Question 5: (*4 pts*) Find the area of the region enclosed by $y = -4x$, $y = 5 - x^2$ and $x = -2$.

Question 6: (*4 pts*) Solve the differential equation $10y' + e^x\,y^3 = 0$, with $y(0) = -1$.

Question 7: (*5 pts*) The weekly demanded quantity x (in hundreds of units) of portable barbecues is related to the price per barbecue p by the equation $p = -0.1x^2 - x + 40$. The supply function for the same product is $p = 0.1x^2 + 2x + 20$.

a) Find the equilibrium point.

b) Sketch and identify the regions representing the consumer and producer surpluses.

c) Evaluate the producer surplus.

Question 8: (*6 pts*) After vacationing in a dubious resort, a group of unicorns have brought back fleas to their herd (which has a total of 5000 unicorns). Initially, 100 unicorns have fleas, but after 10 days the infestation has spread to 511 unicorns. The number of infested unicorns N is increasing at a rate that is proportional to the square root of the number of unicorns that have *not yet* been infested.

a) Write the differential equation (with initial conditions) for the problem.

b) Find the function $N(t)$ for the number of infested unicorns after t days.

c) How long will it take for half of the herd to be infested?

Question 9: (*6 pts*) Evaluate the following limits using l'Hôpital's rule where appropriate.

a) $\displaystyle\lim_{x\to 1} \frac{xe^{x-1} - 3x^2 + 4x - 2}{x^3 + 2x^2 - 7x + 4}$ **b)** $\displaystyle\lim_{x\to +\infty} \frac{\ln(4x+3)}{e^{2x^2} + 5}$

Question 10: (*10 pts*) Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

a) $\displaystyle\int_2^4 \frac{4x-2}{x^2 - x - 12}\,dx$ **b)** $\displaystyle\int_{e^2}^{+\infty} \frac{4}{x\left(\ln(x)+2\right)^2}\,dx$

Question 11: (*2 pts*) Determine whether the sequence $\left\{\dfrac{n^2\,(2n)!}{(2n+2)!}\right\}$ converges or diverges. If the sequence converges, find its limit. If it diverges, explain why.

Question 12: (*2 pts*) True or False: If the sequence $\left\{a_n\right\}$ converges, then the sequence $\left\{(-1)^n\ a_n\right\}$ converges as well. Explain your answer.

Question 13: (*5 pts*) Consider the sequence $\left\{\dfrac{1}{5}, \dfrac{2}{8}, \dfrac{4}{11}, \dfrac{8}{14}, \cdots\right\}$

a) Give the nth term a_n of the sequence.

b) Does the sequence converge? If so, find its limit.

c) Does the series $\displaystyle\sum_{n=1}^{+\infty} a_n$ converge or diverge? State which test you are using.

Question 14: (*4 pts*) A collector wants to buy an autographed Wayne Gretzky rookie card (in mint condition!) valued at \$22 000. If the collector is making weekly deposits in a savings account that earns 2.6% annual interest (compounded weekly), what should the amount of the deposits be in order to have enough money to buy the card after 2 years?

Question 15: (*14 pts*) Determine whether the following series converge or diverge. Identify which test you are using. In the case of a geometric or telescoping series, find the sum of the series.

a) $\displaystyle\sum_{n=3}^{+\infty} \frac{2n+3}{n^3}$ **b)** $\displaystyle\sum_{n=1}^{+\infty} \frac{2}{n^2+4n+3}$ **c)** $\displaystyle\sum_{n=0}^{+\infty} \frac{5^n}{4^{2n+1}\,(2n+1)}$ **d)** $\displaystyle\sum_{n=1}^{+\infty} \frac{3^{-n}}{4^{n-2}}$

ANSWERS:

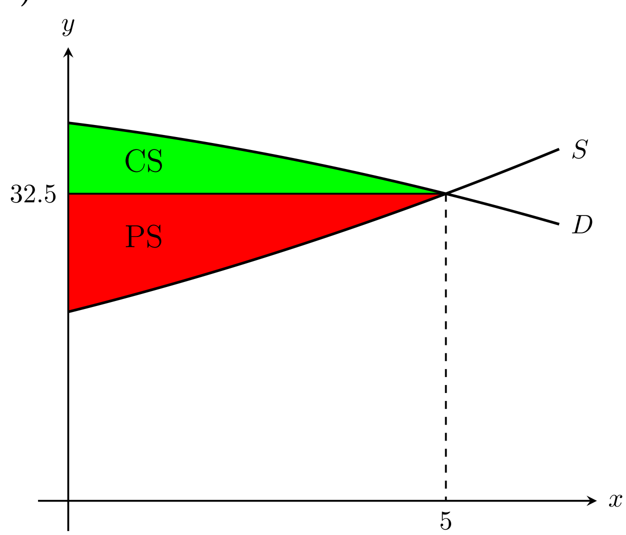
1.) **a)** $\frac{4}{3}t^{3/2} - 4\ln|t| + \frac{1}{2}\ln\Big|\csc(2t) - \cot(2t)\Big| + \frac{10}{\sqrt{t}} + C$ **b)** $\frac{-\ln(x)}{x^2} - \frac{1}{2x^2} + C$

c) $\frac{-7}{8}\sqrt{8 - 4\tan(4x)} + C$ **d)** $\frac{3}{4}(1 - e^2)$ **e)** $2\sec(x) + \cos(x) + 2\ln\Big|\sec(x)\Big| + C$ **f)** $\frac{47}{6}$

g) $\frac{3}{2}x^2 - x + 2\ln|x| - \ln|x+2| + 5\ln|x-3| + C$

2.) $f(x) = x^5 + x^4 + 2x^2 + 8x - 7$ **3.)** 3.4650 **4.)** 43 **5.)** $\frac{10}{3}$ **6.)** $y = -\sqrt{\frac{5}{e^x + 4}}$

7.) **a)** $(5, 32.5)$
b)



c) \$33.33

8.) **a)** $\frac{dN}{dt} = k\sqrt{5000 - N}$

$N = 100$ when $t = 0$, $N = 511$ when $t = 10$

b) $N = 5000 - (70 - 0.3t)^2$

c) Approx. 66.7 days

9.) **a)** $\frac{-3}{10}$ **b)** 0

10.) **a)** Diverges **b)** Converges to 1

11.) Converges to $\frac{1}{4}$

12.) False. It converges only if a_n converges to 0. If a_n converges to a different value, then it will diverge.

13.) **a)** $\frac{2^{n-1}}{3n+2}$ **b)** $\lim_{x \to +\infty} a_n = +\infty$ therefore the sequence diverges.

c) Since $\lim_{x \to +\infty} a_n = +\infty$, the series diverges (by Divergence Test)

14.) \$206.04 per week **15.)** **a)** Converges (sum of 2 p-series) **b)** Converges to $\frac{5}{6}$ (Telescoping)

c) Converges (Ratio Test) **d)** Converges to $\frac{16}{11}$ (Geometric series)