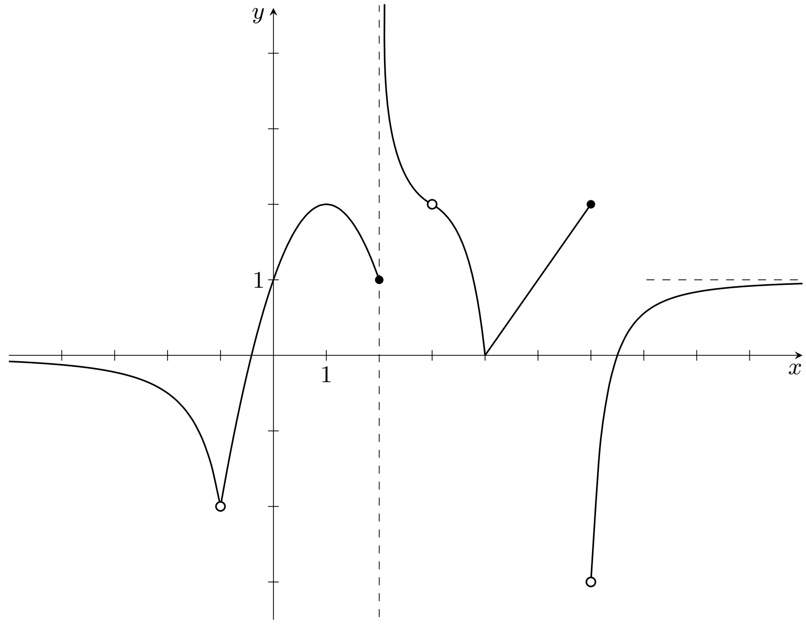
(Marks)

(7) 1. For the function f whose graph is shown below, determine each of the following. Use "does not exist" (DNE), ∞ or $-\infty$, where appropriate.



(a) $\lim_{x\to -1} f(x) =$

(b) $\lim_{x\to 2^-} f(x) =$

(c) $\lim_{x\to 2^+} f(x) =$

(d) $\lim_{x\to 6^+} f(x) =$

(e) $\lim_{x\to 6} f(x) =$

(f) $\lim_{x\to -\infty} f(x) =$

(g) $\lim_{x\to \infty} f(x) =$

(h) $\lim_{x\to 4} \dfrac{1}{f(x)} =$

(i) $f(-1) =$

(j) $f(6) =$

(k) List all $x-$values where the function is discontinuous.

(l) List all $x-$values where the function is continuous but not differentiable.

(18) 2. Evaluate the following limits. Use "does not exist" (DNE), ∞ or $-\infty$, where appropriate.

(a) $\lim_{x\to 1} \dfrac{x^2-3x+2}{3x^2-4x+1}$

(b) $\lim_{x\to 3} \dfrac{x-3}{\frac{1}{x+1}-\frac{1}{7-x}}$

(c) $\lim_{x\to 3} \dfrac{x-3}{\sqrt{x+1}-2}$

(d) $\lim_{x\to 3^+} \dfrac{3x|3-x|}{3-x}$

(e) $\lim_{x\to -\infty} \dfrac{(2x+3)(3x^2-7x+1)}{x^2+1}$

(f) $\lim_{x\to 2} f(x)$ where $f(x) = \begin{cases} x-3 & \text{if} \quad x<2 \\ \frac{x}{2} & \text{if} \quad 2<x \end{cases}$

(4) 3. Use the definition of continuity to determine the points of discontinuity of the following function:

$$f(x) = \begin{cases} \dfrac{x-1}{x^2-2x} & \text{if} \quad x<1 \\ \dfrac{x^2-3x+2}{x^2-x-2} & \text{if} \quad 1\le x\le 3 \\ \dfrac{1}{x+1} & \text{if} \quad 3<x \end{cases}$$

(3) 4. Find the value(s) of a for which the following function is continuous on $\mathbb{R}$. $f(x) = \begin{cases} ax+5 & \text{if} \quad x\le -3 \\ 1-x & \text{if} \quad -3<x \end{cases}$

(Marks)

(5) 5. Use the limit definition of the derivative to calculate the derivative of $f(x) = \sqrt{2x-1}$

(21) 6. Compute $\frac{dy}{dx}$ for each of the following equations. Use properties of logarithms where appropriate. Do NOT simplify your answers.

(a) $y = \frac{2}{x^3} - \sqrt[5]{x^2} - \pi x + \log_3(\cot x) + 6^{2x} - e^2$

(b) $y = \sec(\sqrt{x^4 - 7x})$

(c) $y = 3(e^{2x} - 1)^5(2 - 9x)^4$

(d) $x^2y^3 = \sin(2y)$

(e) $y = \ln\left[\frac{\sqrt{x}\tan^8 x}{(x^2+4)^{10}}\right]$

(f) $y = x^{\cos(3x)}$

(2) 7. Suppose f is a differentiable function such that $f(2) = -3$ and $f'(2) = 4$. If $g(x) = [f(x)]^3$, compute $g'(2)$.

(4) 8. Given $e^{x^2+y} = y\ln x + xy^2$, find an equation of the line tangent to the curve at the point $(1, -1)$.

(4) 9. Find the second derivative of $y = \sqrt{x^2+1}$ and simplify.

(4) 10. Find the absolute extrema of $f(x) = \frac{2x^2-5x+18}{x}$ on the interval $[1, 4]$.

(4) 11. Use the second derivative test to find the local extrema of $f(x) = -3x^5 + 5x^3$. If the test fails, simply state this.

(10) 12. Consider $f(x) = \dfrac{-x}{x^2-4}$, with $f'(x) = \dfrac{x^2+4}{(x^2-4)^2}$, and $f''(x) = \dfrac{-2x(x^2+12)}{(x^2-4)^3}$.

Determine the following, then neatly sketch the graph of $f(x)$ on the following page. Clearly label any important points.

(a) the domain of f,

(b) all vertical and horizontal asymptotes,

(c) all $x-$ and $y-$intercepts,

(d) the intervals on which f is increasing and decreasing,

(e) all local extrema of f,

(f) the intervals on which f is concave up and concave down,

(g) the inflection points of f,

(h) sketch the graph on the next page.

(2) 13. The total weekly cost of production (in dollars) of the BestFit dog harness is $C(x) = -0.05x^2 + 10x + 500$. Find the marginal cost when $x = 50$. Interpret the result.

(4) 14. A gardener wishes to enclose a 24 m^2 rectangular plot of land with a fence. Three sides of the plot will use plain fencing and the side facing the road will use fancy fencing. If the plain fencing costs 10$ per meter and the fancy fencing costs 20$ per meter, find the dimensions of the rectangular plot that will minimize the gardener's cost to put up the fence.

(Marks)

(4) 15. The Mudhouse Café has 5 locations on the Island of Montreal. Every day, each location sells on average 110 cups of coffee. Because business is going so well, they are looking to expand. The manager has estimated that for every new location that opens, the daily number of cups of coffee sold (per location) will decrease by 10. How many new locations should open in order to maximize the total number of cups of coffee sold each day?

(4) 16. Peelocase is a plant-based compostable cell phone case with demand function $x = \frac{1}{80}(2800 - p^2)$.

(a) Find the elasticity of demand function.

(b) Is the demand elastic or inelastic when the price $p = \$40$?

(c) When the price is $p = \$40$, if the price is decreased by 5%, how would the demand be affected?

(d) What price would maximize revenue?

ANSWERS:

1. (a)-2, (b) 1, (c) ∞, (d) -3, (e) DNE, (f) 0, (g) 1, (h) ∞, (i) DNE, (j) 2, (k) -1, 2, 3, 6, (l) 4
2. (a) $-\frac{1}{2}$, (b) -8, (c) 4, (d) -9, (e) $-\infty$, (f) DNE
3. Discontinuity at 0, 2 and 3
4. $a = \frac{1}{3}$
5. $f'(x) = \frac{1}{\sqrt{2x-1}}$
6. (a) $\frac{dy}{dx} = \frac{-6}{x^4} - \frac{2}{5}x^{-3/5} - \pi + \frac{-\csc^2 x}{\cot x \ln 3} + 6^{2x} \cdot 2\ln 6$

   (b) $\frac{dy}{dx} = \sec(\sqrt{x^4 - 7x})\tan(\sqrt{x^4 - 7x})\frac{1}{2\sqrt{x^4-7x}}(4x^3 - 7)$

   (c) $\frac{dy}{dx} = 15(e^{2x} - 1)^4(2e^{2x})(2 - 9x)^4 + 12(e^{2x} - 1)^5(2 - 9x)^3(-9)$

   (d) $\frac{dy}{dx} = \frac{2xy^3}{2\cos(2y) - 3x^2y^2}$

   (e) $\frac{dy}{dx} = \frac{1}{2x} + \frac{8\sec^2 x}{\tan x} = \frac{10}{x^2 + 4}(2x)$

   (f) $\frac{dy}{dx} = x^{\cos(3x)}[\frac{\cos(3x)}{x} - 3\sin(3x)\ln x]$
7. 108
8. $y = -\frac{2}{3}x - \frac{1}{3}$
9. $y" = \frac{1}{(x^2+1)^{3/2}}$
10. absolute max. = 15 at $x = 1$, absolute min. = 7 at $x = 3$
11. local max. = 2 at $x = 1$, local min. = -2 at $x = 1$, test fails at $x = 0$

(Marks)

12.

(a) $x \neq \pm 2$,

(b) V.A. $x = \pm 2$, H.A. $y = 0$,

(c) $x-$ and $y-$intercept $(0, 0)$,

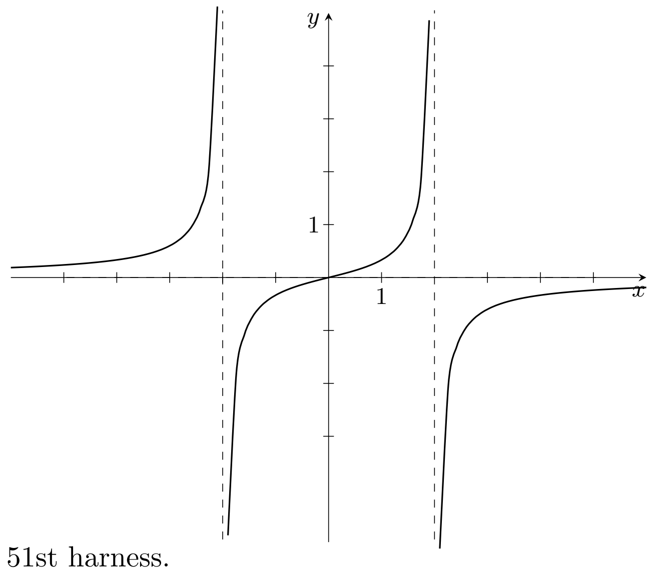
(d) increasing on $(-\infty, -2), (-2, 2)$ and $(2, \infty)$,

(e) no local extrema,

(f) concave up on $(-\infty, -2)$ and $(0, 2)$,
concave down on $(-2, 0)$ and $(2, \infty)$,

(g) $(0, 0)$

(h)



13. Marginal cost $C'(50) = 5\$$ is the cost of producing 51st harness.

14. 6 m (facing the road) by 4m

15. 3 new locations should open.

16. (a) $E(p) = \frac{2p^2}{2800-p^2}$, (b)elastic ($E(40) > 1$), (c)$\frac{40}{3}\% \approx 13\%$, (d)$\sqrt{\frac{2800}{3}}$ ≈30.55$