1. Evaluate the following integrals.

(5) (a) $\displaystyle\int \frac{3x^2+7x-3}{x^2(x-3)}\,dx$

(5) (b) $\displaystyle\int x\sin x\cos^2 x\,dx$

(5) (c) $\displaystyle\int x^5\sqrt{x^3+1}\,dx$

(5) (d) $\displaystyle\int_{-3}^{1} \frac{3x}{\sqrt{55-x^2-6x}}\,dx$

(5) (e) $\displaystyle\int \frac{dx}{x\ \left((\ln x)^2-25\right)^{3/2}}$

(5) (f) $\displaystyle\int \sqrt{1+e^x}\,dx$

2. Evaluate the following limits.

(3) (a) $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$

(3) (b) $\displaystyle\lim_{x\to \pi^+}\left(\frac{2x-\pi}{\pi}\right)^{2\csc(x)}$

3. Evaluate each improper integral or show it diverges.

(4) (a) $\displaystyle\int_{3}^{5} \frac{x}{\sqrt{x^2-9}}\,dx$

(4) (b) $\displaystyle\int_{1}^{\infty} x^2\ln(x^2)\,dx$

4. Let $\mathcal{R}$ be the region bounded $y=x^3$ and $y=4x$. Set up, but **do not evaluate**, the integral for the volume of the solid obtained by rotating $\mathcal{R}$ around

(3) (a) the region $\mathcal{R}$ about the $x-$axis

(3) (b) the region $\mathcal{R}$ about the line $x=3$

(4) **5.** Set up, **but do not evaluate**, the integral(s) for the area of the region enclosed by the functions $x=y^4$ and $3y^2=x-4$ when $y\geq 0$.

(5) **6.** Find the solution of the differential equation that satisfies the given initial condition.

$$\frac{dy}{dx}=\frac{xe^{x^2}}{y^2} \qquad \text{given } y(0)=1$$

(4) **7.** The Springfield pond has a volume of 10 ML (megalitre) of pure, clear water. Unfortunately, Mr. Burns begins dumping contaminated water, containing 20 g per ML of plutonium, at a rate of 5 ML per year. In order to not get caught, he also drains the pond at the same rate.

Assuming the pond is thoroughly mixed (and all fish are fine), find the equation representing the amount of plutonium in the lake as a function of time.

8. Determine whether the following sequences converge or diverge. In the case of convergence, find the limit.

(2) (a) $\left\{ \dfrac{n^2(3n-1)!}{(3n+1)!} \right\}$

(2) (b) $\left\{ n \sin\left(\dfrac{1}{n}\right) \right\}$

9. Suppose that $\sum_{n=1}^{\infty} a_n$ is a a series with the partial sum formula $s_n = \left(e^{2/n} - 1\right)$ Find

(2) (a) $\sum_{n=1}^{\infty} a_n$

(2) (b) $\lim_{n\to\infty} a_n$

(1) (c) $\sum_{n=4}^{\infty} a_n$

10. Determine whether the following series converge or diverge. Justify your answers.

(2) (a) $\sum_{n=1}^{\infty} \dfrac{e^n + 7^n}{n^2}$

(2) (b) $\sum_{n=1}^{\infty} \dfrac{n+5}{5^n}$

(2) (c) $\sum_{n=1}^{\infty} \dfrac{3\cos^2(n)}{\sqrt[3]{n+n^5}}$

(2) (d) $\sum_{n=1}^{\infty} \dfrac{7+3n}{\sqrt{5n^3+3}}$

11. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Justify your answers.

(3) (a) $\sum_{n=1}^{\infty} (-1)^n \dfrac{2n+1}{n^{3/2}}$

(3) (b) $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}(5n)5^n}{(5n)!}$

(3) (c) $\sum_{n=1}^{\infty} (-1)^n \left(\dfrac{n}{n+1}\right)^{n^2}$

(5) **12.** Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{(x-1)^n}{5^n\sqrt{3n-1}}$$

(4) **13.** Find the Maclaurin series of
$g(x) = xe^x$.

(2) **14.** If $\sum a_n$ and $\sum b_n$ are convergent series with positive terms, prove or disprove that $\sum a_n b_n$ is convergent.

Answers:

1. (a) $-2\ln|x| - \frac{1}{x} + 5\ln|x-3| + c$

(b) $\frac{-x}{3}\cos^3 x + \frac{1}{3}(\sin x - \frac{1}{3}\sin^3 x) + c$

(c) $\frac{1}{3}\left(\frac{2}{5}(x^3+1)^{(5/2)} - \frac{2}{3}(x^3+2)^{(3/2)}\right) + c$

(d) $-12\sqrt{3} + 24 - \frac{3\pi}{2}$, indefinte is $3\left(-\sqrt{64-(x+3)^2} - 3\arcsin\left(\frac{x+3}{8}\right)\right) + c$

(e) $\dfrac{-1}{25}\dfrac{\ln x}{\sqrt{(\ln x)^2 - 25}} + c$

(f) $2\sqrt{1+e^x} + \ln\left(\sqrt{1+e^x} - 1\right) - \ln\left(\sqrt{1+e^x} + 1\right) + c$

2. (a) $\frac{-1}{2}$

(b) $e^{-\frac{4}{\pi}}$

3. (a) 4

(b) diverges

4. (a) $\displaystyle\int_{-2}^{2} \pi(4x)^2 - \pi(x^3)^2\, dx$

(b) $\displaystyle\int_{-2}^{0} 2\pi(3-x)(x^3-4x)\, dx + \int_{0}^{2} 2\pi(3-x)(4x-x^3)\, dx$

5. $\displaystyle\int_{0}^{2} 3y^2 + 4 - y^4\, dy$

6. $y = \sqrt[3]{\dfrac{3}{2}e^{x^2} - \dfrac{1}{2}}$

7. $y = 200 - 200e^{\frac{-t}{2}}$

8. (a) $\frac{1}{9}$

(b) 1

9. (a) 0

(b) 0

(c) $1 - e^{2/3}$

10. (a) Diverges, Diverges Test

(b) Converges, Ratio Test

(c) Converges, Comparison Test

(d) Diverges, Limit Comparison Test

11. (a) Conditional Convergence

(b) Absolute Convergence

(c) Absolute Convergence

12. R = 5, I = $(-4, 6]$

13. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{(n-1)!}$

14. It can be proven in various ways. If $\sum a_n = A$ since converges, then $a_n \leq A$ since all a_n is positive. Then $\sum a_n b_n \leq \sum A b_n = A\sum b_n$, which converges, so by comparison test $\sum a_n b_n$ converges.