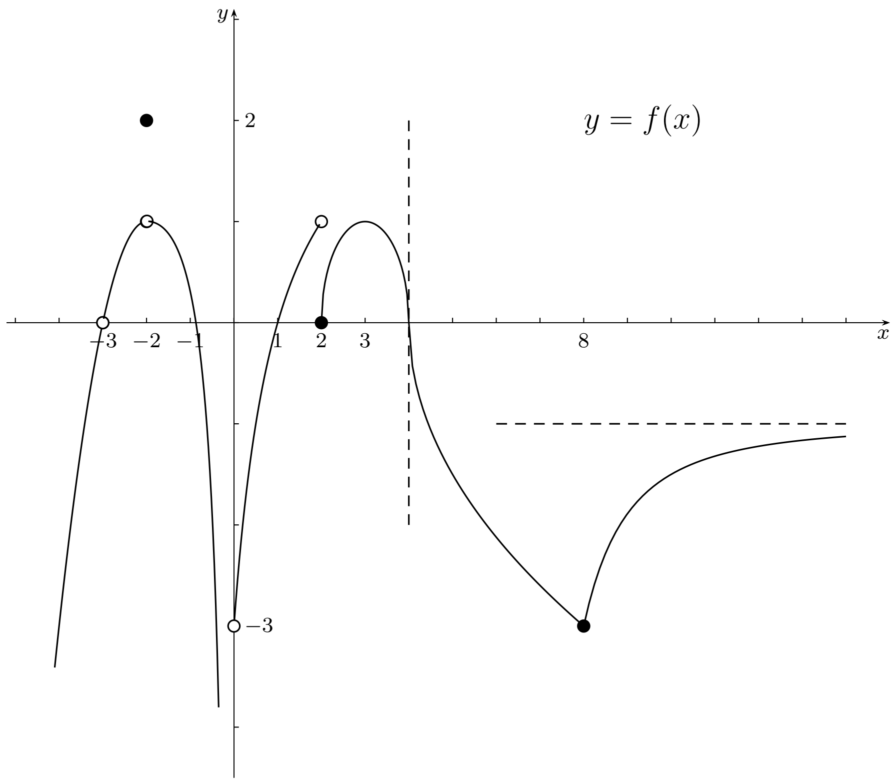
(7) 1. Given the graph of f below, determine each of the following. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate. (Note that there is a vertical tangent line at $x = 4$).



(a) $\lim_{x\to-\infty} f(x) =$

(b) $\lim_{x\to-2} f(x) =$

(c) $\lim_{x\to0^-} f(x) =$

(d) $\lim_{x\to0^+} f(x) =$

(e) $\lim_{x\to2^-} f(x) =$

(f) $\lim_{x\to2^+} f(x) =$

(g) $\lim_{x\to\infty} f(x) =$

(h) $f(-2) =$

(i) $f'(3) =$

(j) List all x-values where the function is discontinuous.

(k) List all x-values where the function is continuous but not differentiable.

(18) 2. Evaluate the following limits. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.

(a) $\lim_{x\to-1} \dfrac{x^2-1}{x^2-2x-3}$

(b) $\lim_{x\to3} \dfrac{\frac{1}{3}-\frac{1}{x}}{(3-x)}$

Question 2 continues on the next page.

(c) $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x+2}-\sqrt{2x}}$

(d) $\lim_{x\to -1} \dfrac{x-3}{|x+1|}$

(e) $\lim_{x\to \infty} \dfrac{(3x+1)^2(x^2+1)^3}{(x^3-1)^2(x+1)^2}$

(f) $\lim_{x\to -\infty} \dfrac{x^3-1}{x^2+1}$

(4) 3. Use the definition of continuity to determine the points of discontinuity of the following function.

$$f = \begin{cases} \dfrac{x^2+2x}{x^2-4} & x \le 0 \\ x & 0 < x < 3 \\ \dfrac{x}{x-5} & x \ge 3 \end{cases}$$

(3) 4. Find the value(s) of k for which the following function is continuous on $\mathbb{R}$.

$$g = \begin{cases} k^2+3x & x < 1 \\ 3+5kx & x \ge 1 \end{cases}$$

(5) 5. (a) State the limit definition of the derivative, $f'(x)$.

(b) Use this definition to calculate the derivative of $f(x) = \dfrac{2}{3-2x}$

(20) 6. For of each of the following, find y'. Do not simplify your answers.

(a) $y = \pi^3 + 3^{\pi x} - x^3 + \log_3 \pi - (\ln 3)x - \frac{3}{\pi x}$

(b) $y = \tan^3(e^x - x^e)$

(c) $y = \dfrac{3x - \cot(3x)}{1 + \csc(3x)}$

(d) $y = \sqrt{\cos(3x^2+1) + x}$

(e) $\ln(xy) - y^2 = 5$

(f) $y = (\sin x)^{\ln x}$

(4) 7. Use logarithmic differentiation to find the derivative of $y = \dfrac{(2t+5)^3 e^{5t}}{7\tan^2 t}$

(4) 8. Given $(x+y)^2 + x^2y = 11$, find an equation of the line tangent to the curve at the point $(1, 2)$.

(4) 9. Find the 2[nd] derivative of $g(x) = \dfrac{16x^3 - 2\sqrt{x} + 8x^2 - 4x^2e^{3x}}{4x^2}$

(4) 10. Use the second derivative test to find the local extrema of $f(x) = x(x-1)^3$.
If the test fails, simply state this.

(4) 11. Find the absolute extrema of $f(x) = \sqrt[3]{x^2 - 4x}$ on the interval $[-1, 3]$.

(10) 12. Consider $f(x) = \dfrac{x^2 - 1}{x^2 - 4}$ with $f'(x) = \dfrac{-6x}{(x^2 - 4)^2}$ and $f''(x) = \dfrac{6(3x^2 + 4)}{(x^2 - 4)^3}$.
Determine the following, then neatly sketch the graph of $f(x)$ on the following page.

(a) all x- and y- intercepts;

(b) all vertical and horizontal asymptotes;

(c) the intervals on which $f(x)$ is increasing and decreasing;

(d) all local (relative) maxima and minima;

(e) the intervals on which $f(x)$ is concave up and concave down;

(f) any points of inflection;

(g) sketch the curve $y = f(x)$ on the grid below.

(4) 13. Suppose a gaming console has a weekly demand function given by $p = 220 - 0.02x$ and a weekly cost function given by $C(x) = 0.0001x^3 + 0.01x^2 + 31x + 41500$, where x is the number of gaming consoles.

(a) Find the production level that would maximize weekly profit.

(b) At what price should the gaming consoles be sold in order to maximize weekly profits?

Question 13 continues on the next page.

(4) 14. Shawn is trying to raise money for his trip to Cambodia, to help with repairs to an orphanage. He has discovered from former experience that 33 people will donate if he asks for donations of \$100. However, everytime he lowers the requested amount by \$4, three more people donate.

(a) How much should he ask people to donate in order to maximize the total donations?

(b) What is the maximum of the total donations?

(5) 15. The demand function of a Rubik's cube is given by $x = \sqrt{11367 - 108p^2}$.

(a) Find the elasticity of demand function.

(b) Is the demand elastic or inelastic when $p = \$5.50$?

(c) When the price is $p = \$5.50$, if the price is increased by 3%, how would the demand be affected?

(d) What price would maximize revenue?

Solution:

1. a) $-\infty$; b)1; c)$-\infty$; d)-3; e)1; f)0; g)-1; h)2; i)0; j)-3,-2, 0, 2; k)4, 8

2. a)$\frac{1}{2}$; b)$-\frac{1}{9}$; c)-4; d)$-\infty$; e)9; f)$-\infty$

3. $x = -2$ since $f(-2)$ is undefined; $x = 3$ since the limit does not exist; $x = 5$ since $f(5)$ is undefined.

4. $k = 0, 5$

5. (a) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

   (b) $f'(x) = \dfrac{4}{(3-2x)^2}$

6. (a) $y' = (\pi \ln 3)3^{\pi x} - 3x^2 - \ln(3) + \frac{3}{\pi x^2}$

   (b) $y' = 3\tan^2(e^x - x^e)\sec^2(e^x - x^e)(e^x - ex^{e-1})$

   (c) $y' = \dfrac{[3 + 3\csc^2(3x)][1 + \csc(3x)] + [3x - \cot(3x)][3\csc(3x)\cot(3x))}{[1 + \csc(3x)]^2}$

   (d) $y' = \dfrac{-\sin(3x^2+1)(6x) + 1}{2\sqrt{\cos(3x^2+1) + x}}$

   (e) $y' = \dfrac{\frac{1}{x}}{2y - \frac{1}{y}}$

   (f) $y' = (\sin x)^{\ln x}[\frac{\ln(\sin x)}{x} + \ln x \cot x]$

7. $f'(x) = \dfrac{(2t+5)^3 e^{5t}}{7\tan^2 t}\left[\dfrac{6}{2t+5} + 5 - \dfrac{2\sec^2 t}{\tan t}\right]$

8. $y = -\frac{10}{7}x + \frac{24}{7}$

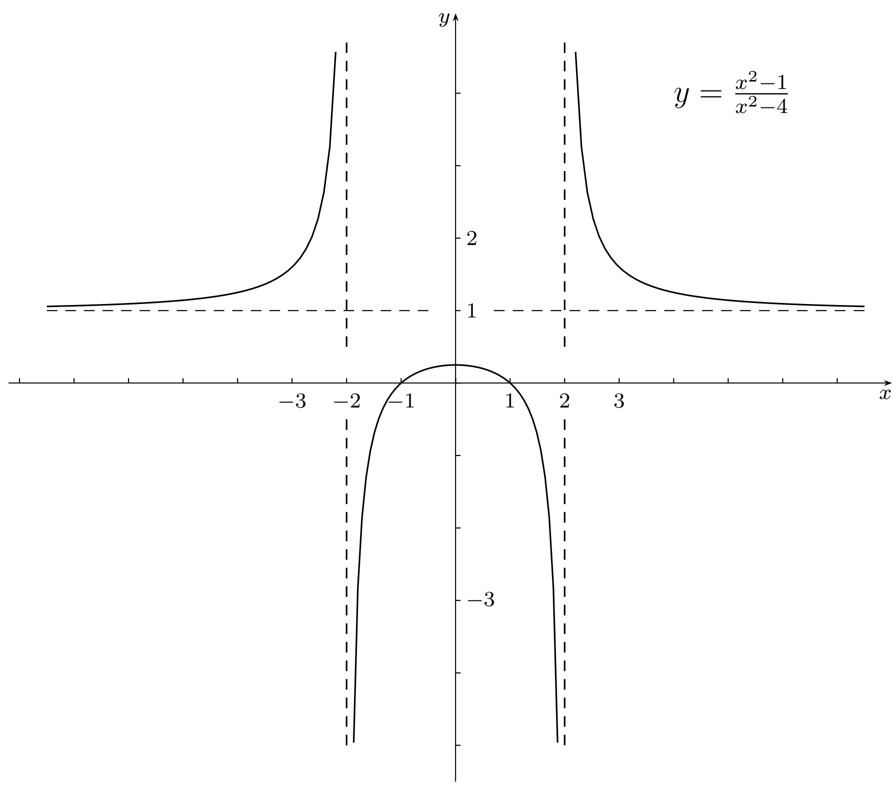
9. $f''(x) = -\frac{15}{8}x^{-\frac{7}{2}} - 9e^{3x}$.

Question 15 continues on the next page.

10. Local minimum at $(\frac{1}{4}, -\frac{27}{256})$ Test fails at $x = 1$.

11. absolute max at $x = -1$, absolute min at $x = 2$

12. (a) x-intercepts at $(1, 0), (-1, 0)$; y- intercept at $(0, \frac{1}{4})$.
    (b) Vertical asymptotes at $x = -2, x = 2$, horizontal asymptote at $y = 1$: $y \to 1$ as $x \to \pm\infty$
    (c) Increasing on $(-\infty, -2), (-2, 0)$ and decreasing on $(0, 2), (2, \infty)$
    (d) Local max at $(0, \frac{1}{4})$
    (e) Concave up on $(-\infty, -2), (2, \infty)$ and concave down on $(-2, 2)$
    (f) No inflection points.
    (g)



13. (a) 700 units
    (b) \$206

14. (a) \$72
    (b) \$3888

15. (a) $\eta = E(p) = \dfrac{4p^2}{421 - 4p^2}$
    (b) Inelastic.
    (c) Demand will decrease by 1.21%
    (d) \$7.25

This is the end of the examination.