1. The game of Scrabble has a bag of letter tiles containing 56 consonants and 44 vowels. Assume all tiles are distinguishable.

(a) (3 points) If 5 tiles are chosen from the bag one by one, **without replacement**, in how many different ways can you pick 3 consonants in a row, followed by 2 vowels?

(b) (3 points) If 5 tiles are chosen from the bag one by one, **with replacement**, in how many different ways can you pick 3 consonants in a row, followed by 2 vowels?

(c) (3 points) If 5 letters are chosen from the bag all at once, what is the probability that 3 are consonants and the remaining 2 are vowels? (Order here is not important.)

2. The table below summarizes information about travellers at Trudeau airport in the North American terminal on a Monday morning. Travellers were asked their destination and the purpose of their travel.

	Canada	USA	Mexico	Total
Business	341	237	151	729
Family	145	79	125	349
Leisure	211	189	177	577
Total	697	505	453	1655

(a) (2 points) Find the probability that a randomly selected traveller will be going to Mexico.

(b) (2 points) Find the probability that a randomly selected traveller is going to the USA, given that he is travelling for business.

(c) (2 points) Find the probability that a randomly selected traveller is travelling for leisure *or* is going to a Canadian destination.

(d) (3 points) Are the events "travelling for family reasons" and "travelling to Canada" independent? Justify your answer.

3. (6 points) Oscar drives to school 2 out of 10 days, and takes the bus otherwise. When he drives, he grabs a coffee at Bob Morton's 70% of the time, but when he takes the bus, he only takes the time to buy a coffee 40% of the time.

One morning, Oscar's teacher notices that Oscar comes to class with a coffee. Given that information, what is the probability that Oscar drove to school?

4. Bob Morton's is holding their annual "Flip your cup over to win" contest, where each coffee cup may have you win another coffee or donut (value 1$), a sandwich (value 5$) or a gift card (value 100$). The following table of probabilities is provided by Bob Morton's:

Prize (Value)	Try again $0	Coffee or Donut $1	Sandwich $5	Gift Card $100
Probability	?	0.1	0.04	0.001

(a) (1 point) Calculate the probability of not getting any prize ("Try again").

(b) (3 points) Calculate the expected value of the prize.

(c) (3 points) Calculate the standard deviation of the prize.

5. Suppose that 4% of all students in CÉGEP are unable to finish their semester because of a medical problem. At the beginning of a semester, 30 students are registered in a given class.

(a) (3 points) Find the probability that exactly 4 students in this class will not finish the semester because of a medical problem.

(b) (5 points) Find the probability that at least 2 students in this class will not finish the semester because of a medical problem.

6. In a population of 29 year-old males, the annual salary follows a normal distribution with a mean salary of \$29,321 with a standard deviation of \$2,120.

(a) (4 points) What is the probability that a man chosen at random earns a salary between \$32,000 and \$36,000?

(b) (4 points) If a sample of 100 men is taken, what is the probability the mean of their salaries will be less than \$29,000?

7. (7 points) In a small village in Africa, the probability that a baby is born with a specific kind of birth defect is 0.03. In the next year, suppose there will be 3000 births. Use a normal approximation to estimate the probability that more than 65 babies will be born with this birth defect in the next year.

8. (5 points) A *ruler drop test* is a method used to measure the reaction time, an important attribute for athletes and race car drivers for example. A ruler is held vertically in the air and released at a random time; the subject needs to catch the ruler between the index finger and the thumb as quickly as possible after the ruler starts falling. The reaction time is then measured as the length (in cm) of the ruler that passed between the subject's fingers before being caught.

A researcher wishes to estimate the true average reaction time of hockey goaltenders by constructing a 92% confidence interval. Assuming that the population standard deviation in the results of this test is 4.4 cm, how many goaltenders should be sampled in order to estimate the average reaction time within 0.5 cm?

9. The number of neurons was estimated in the brains of a random sample of 12 adult fruit flies. In this sample, the data led to an average number of neurons of $253,000$ per fly, with a standard deviation of $17,000$ neurons. Assume that the distribution of the number of neurons in fruit flies follows a normal distribution.

(a) (5 points) Find a 98% confidence interval for the true average number of neurons in fruit flies. (You may round your answer to the nearest neuron.)

(b) (2 points) Carefully interpret your answer from part (a) in the context of this application.

10. A recent poll has shown that 60% of Americans believe that the main stream media reports fake news. Suppose that a similar question is asked to Canadians citizens: in a random sample of 300 Canadians, 152 of them believe the media is reporting fake news. Using a 1% significance level, test the hypothesis that the percentage of all Canadians who believe the media is reporting fake news is lower than among Americans (60%).

(a) (2 points) State the null and alternative hypotheses.

(b) (3 points) Calculate the test statistic.

(c) (3 points) Find (or estimate) the p-value for the test.

(d) (2 points) State and **interpret** your conclusion.

11. A sample of eight nurses work on a rotating system of some night shifts and some day shifts. They were asked about their level of tension after working their shifts. The results are recorded below. A higher number indicates a higher level of stress.

Nurse	1	2	3	4	5	6	7	8
Day Shift	1.5	3	2	3	2	2	1	2
Night shift	3.5	2	4	4	3.5	2	3.5	3

Suppose that stress levels are normally distributed. The head nurse claims that there is no difference in the stress levels between the day and night shifts. Test the alternative hypothesis that there is in fact a significant difference between the stress levels between working the day or night shift, at the 5% significance level.

(a) (2 points) State the null and alternative hypotheses.

(b) (3 points) Calculate the test statistic.

(c) (3 points) Find (or estimate) the p-value for the test.

(d) (1 point) State your conclusion.

12. Does attending class influence how students perform on an exam? For a class of 60 students. the following information was recorded.

	Pass	Fail
Attended	25	5
Skipped	15	15

Test the hypothesis at the 0.5% level of significance that attending class does in fact influence student performance on an exam.

(a) (2 points) State the null and alternative hypotheses.

(b) (3 points) Calculate the test statistic.

(c) (3 points) Find (or estimate) the p-value for the test.

(d) (2 points) State and **interpret** your conclusion.

13. (5 points) A group of students ran a mile. They were classified as either athletes or non-athletes, and their times were recorded in minutes. The summary is given below:

	Sample Size	Sample Average Time	Population Standard Deviation
Non-Athletes	126	8.06	2.02
Athletes	64	7.51	1.41

Find a 99% confidence interval for the true average difference between the time it takes athletes and non-athletes to run a mile.

ANSWERS

1. (a) $P_{56,3} \cdot P_{44,2} = 56 \cdot 55 \cdot 54 \cdot 44 \cdot 43 = 314,677,440$
   (b) $56^3 \cdot 44^3 = 339,992,576$
   (c) $\dfrac{C_{56,3} \cdot C_{44,2}}{C_{100,5}} = 0.3483$

2. (a) $\dfrac{453}{1655} = 0.2737$
   (b) $\dfrac{237}{729} = 0.3251$
   (c) $\dfrac{577 + 697 - 211}{1655} = \dfrac{1063}{1655} = 0.6423$
   (d) $P(\text{family} \mid \text{Canada}) = \dfrac{145}{697} = 0.2080$
   $P(\text{family}) = \dfrac{349}{1655} = 0.2109$
   The above two probabilities are not equal, therefore the events are not independent.

3. 0.3043

4. (a) 0.859 (b) 0.4 (c) 3.3076

5. (a) 0.0243 (b) 0.3388

6. (a) 0.1030 (b) 0.0655

7. 0.9956

8. at least 238 goaltenders

9. (a) (239661 , 266339)
   (b) We are 98% confident that the true average number of neurons among all adult fruit flies is between 239661 and 266339.

10. (a) $H_o : p = 0.60$ $H_a : p < 0.60$
    (b) $z^* = -3.30$
    (c) p-value $= 0.0005$
    (d) reject H_o
    There is sufficient evidence to claim that the true proportion of all Canadians who believe the media is reporting fake news is less than 0.60, at the 1% significance level.

11. (a) $H_o : \mu_d = 0$ $H_a : \mu_d \neq 0$
    (b) $t^* = -2.7495$
    (c) $0.02 < p$-value < 0.05
    (d) reject H_o

12. (a) H_o : attendance and passing the course are independent
    H_a : attendance and passing the course are dependent
    (b) $\chi^{2,*} = 7.5$
    (c) $0.005 < p$-value < 0.01
    (d) fail to reject H_o
    There is insufficient evidence to claim that attendance and passing the course are dependent variables, at the 0.5% significance level.

13. (-0.0999 , 1.1999)