Question 1: (*9 pts*) Evaluate the following limits:

a) $\lim\limits_{x\to 0} \dfrac{e^{2x}+x^2-2x-1}{\cos(3x)-1}$ **b)** $\lim\limits_{x\to+\infty} \left(3x-1\right)^{\frac{2}{\ln(x)+1}}$ **c)** $\lim\limits_{x\to\pi} \sin(2x)\csc(5x)$

Question 2: (*4 pts*) Use Newton's method to find the solution of $\sin(x) = x^2 - 2$ that is between $x = 1$ and $x = 2$. Give an answer that is accurate to 5 decimals. (Show all your work!)

Question 3: (*9 pts*) Find the derivative of the following functions. **Do not simplify your answer.**

a) $y = e^{\arccos(3x)}$ **b)** $y = \left(\arctan(\sqrt{x})\right)^4$ **c)** $y = \ln\left(\arcsin(x^3)\right)$

Question 4: (*4 pts*) Use the Trapezoidal Rule to approximate $\int_1^2 \sqrt{x^3-1}\,dx$, using $n = 5$.

Give an answer that is accurate to 4 decimals.

Question 5: (*4 pts*) For raising a load, the efficiency of a screw with square threads is given by:

$$E = \frac{100T\,(1-fT)}{T+f}$$

where f is the coefficient of friction, and T is the *tangent* of the pitch angle of the screw. If $f = 0.25$, what acute angle will give the maximum efficiency? $\left(\text{Hint: find } \dfrac{dE}{dT}\right)$

Question 6: (*3 pts*) A conveyor belt is dumping gravel at a rate of 40.0 ft^3/min. The gravel is accumulating in a pile that has a conical shape, and whose height and diameter are always equal. How fast is the height of the pile increasing when it's 5.00 ft? $\left(\text{Remember volume of a cone: } V = \dfrac{\pi}{3}r^2h\right)$

Question 7: (*13 pts*)

Given $f(x) = \dfrac{x-1}{x^2-2x}$, $f'(x) = \dfrac{-(x^2-2x+2)}{(x^2-2x)^2}$ and $f''(x) = \dfrac{2(x-1)(x^2-2x+4)}{(x^2-2x)^3}$, find (if any):

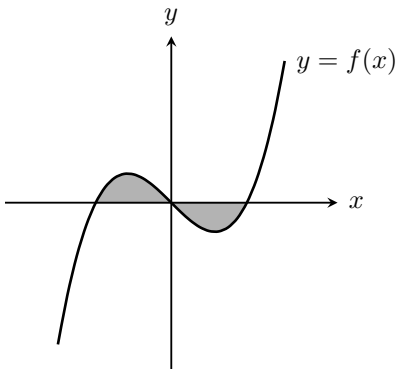
a) The domain of f. **b)** The x and y intercept(s). **c)** The vertical and horizontal asymptotes.

d) The intervals on which f is increasing or decreasing. **e)** The local minima and maxima.

f) The intervals of upward and downward concavity. **g)** The points of inflection.

h) Sketch the graph of f.

Question 8: (*3 pts*) Find the area of the region enclosed by the graph of $f(x) = x^3 - x$ and the x-axis:



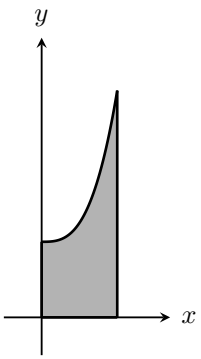
Question 9: (*28 pts*) Evaluate the following integrals:

a) $\displaystyle\int \frac{2x^3 - 2x^2 - 6x + 13}{x^2 - x - 2}\, dx$ **b)** $\displaystyle\int \tan^3(x)\, \cos^2(x)\, dx$ **c)** $\displaystyle\int \arccos(x)\, dx$

d) $\displaystyle\int \sqrt{x^2 + 6x + 10}\, dx$ **e)** $\displaystyle\int \frac{x^2 + 1}{\sqrt[5]{x^3 + 3x + 7}}\, dx$ **f)** $\displaystyle\int \tan^8(3x)\, \sec^4(3x)\, dx$

g) $\displaystyle\int \left(3x^2 + 2\right) e^{2x}\, dx$

Question 10: (*6 pts*) Let R be the region enclosed by $y = 1 + 2x^3$, $y = 0$, $x = 0$ and $x = 1$:



[2] **a)** Set up, but **do not evaluate**, the integral to find the volume of the solid obtained by revolving R about the line $y = -2$.

[4] **b)** Find the volume of the solid obtained by revolving R about the y-axis.

Question 11: (*3 pts*) Determine if $y = x^3e^{2x}$ is a solution of the differential equation $y''-4y'+6y = 6xe^{2x}$.

Question 12: (*4 pts*) Solve the following separable differential equation: $2\sin(x)\dfrac{dy}{dx} = \dfrac{\tan(x)}{y}$ with the initial condition $y(0) = -2$.

Question 13: (*4 pts*) Solve the following first order linear differential equation:

$$x\,y' \;=\; y \;+\; \frac{2x^2}{x^2+1} \quad \text{with initial condition } \; y(\pi/4) \;=\; \pi$$

Question 14: (*6 pts*) Given the function $f(x) = \begin{cases} 0 & \text{if} \quad -\pi \leqslant x < 0 \\ x & \text{if} \quad 0 \leqslant x < \pi \end{cases}$ find a_0, a_1 and b_1 of the Fourier series of the function.

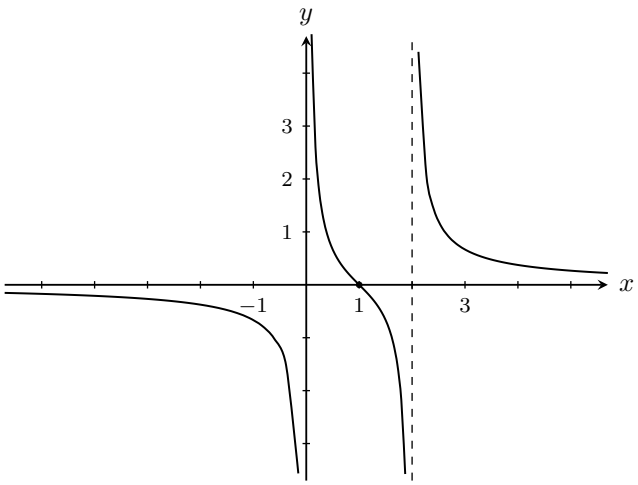
ANSWERS:

1.) **a)** $\dfrac{-2}{3}$ **b)** e^2 **c)** $\dfrac{-2}{5}$ **2.)** 1.72847 **3.)** **a)** $e^{\arccos(3x)} \cdot \dfrac{1}{\sqrt{1-(3x)^2}} \cdot 3$

b) $4\Big(\arctan(\sqrt{x})\Big)^3 \cdot \dfrac{1}{1+(\sqrt{x})^2} \cdot \dfrac{1}{2\sqrt{x}}$ **c)** $\dfrac{1}{\arcsin(x^3)} \cdot \dfrac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2$ **4.)** 1.4909

5.) 0.6629rad or 37.98° **6.)** 2.04 ft/min

7.) **a)** $\mathbb{R}\backslash\{0,2\}$ **b)** x-int: $(1,0)$ y-int: none
c) V.A.: $x=0$ and $x=2$ H.A.: $y=0$
d) Inc. never, Dec. on $]-\infty,0[$, $]0,2[$ and $]2,+\infty[$
e) None
f) C.U.: $]0,1[$ and $]2,+\infty[$ C.D.: $]-\infty,0[$ and $]1,2[$ **g)** (1,0)



8.) $\dfrac{1}{2}$ **9.)** **a)** $x^2+3\ln|x-2|-5\ln|x+1|+C$ **b)** $\dfrac{\cos^2(x)}{2}-\ln|\cos(x)|+C$

c) $x\arccos(x)-\sqrt{1-x^2}+C$ **d)** $\dfrac{1}{2}(x+3)\sqrt{x^2+6x+10}+\dfrac{1}{2}\ln\left|\sqrt{x^2+6x+10}+x+3\right|+C$

e) $\dfrac{5}{12}\left(x^3+3x+7\right)^{4/5}+C$ **f)** $\dfrac{1}{33}\tan^{11}(3x)+\dfrac{1}{27}\tan^9(3x)+C$

g) $\dfrac{1}{2}(3x^2+2)e^{2x}-\dfrac{3}{2}xe^{2x}+\dfrac{3}{4}e^{2x}+C$ **10.)** **a)** $\displaystyle\int_0^1 \pi\Big[\left(3+2x^3\right)^2-2^2\Big]\,dx$ **b)** $\dfrac{9\pi}{5}$

11.) It is not a solution **12.)** $y=-\sqrt{\ln\left|\sec(x)\tan(x)\right|+4}$ **13.)** $y=2x\arctan(x)+2x$

14.) $a_0=\dfrac{\pi}{4}$ $a_1=\dfrac{-2}{\pi}$ $b_1=1$