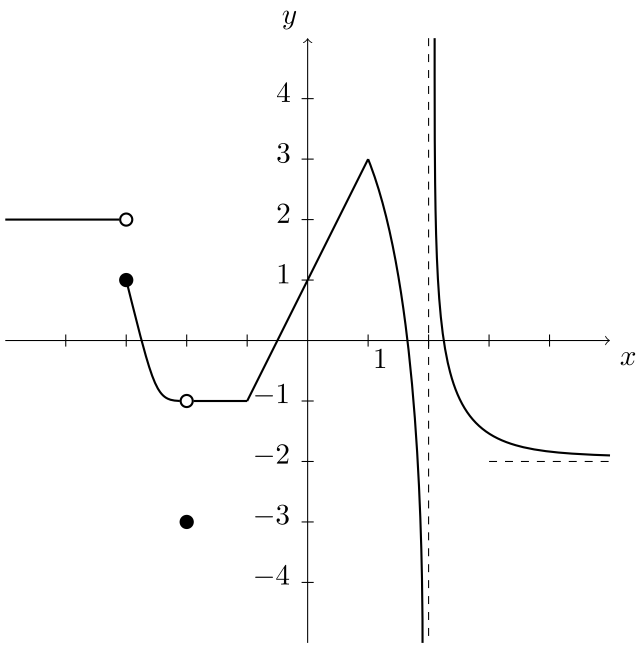
(6) **1.** Given the graph of f below, determine each of the following. Use ∞, $-\infty$ or "does not exist" (DNE) where appropriate.



(a) $\lim_{x\to -2} f(x) =$

(b) $\lim_{x\to -3^+} f(x) =$

(c) $f(-2) =$

(d) $f'(0) =$

(e) $\lim_{x\to \infty} f(x) =$

(f) $\lim_{x\to 2^-} f(x) =$

(g) $\lim_{x\to -\infty} f(x) =$

(h) $\lim_{x\to 2} f(x) =$

(i) $f'(1) =$

(j) List all x-values where the function is discontinuous and name the types of discontinuity.

(k) List all x-values where the function is continuous but not differentiable. Justify your answer.

(21) **2.** Evaluate the following limits. Use ∞, $-\infty$, and dne as appropriate.

(a) $\lim_{x\to 3} \dfrac{\frac{3}{7} - \frac{x}{3x-2}}{x-3}$

(b) $\lim_{x\to 4} \dfrac{\sqrt{3x} - \sqrt{4x-4}}{2x-8}$

(c) $\lim_{x\to 10} \dfrac{x^2 + x - 110}{x+11}$

(d) $\lim_{x\to 1} \dfrac{5x^2 - 5}{15x^2 - 18x + 3}$

(e) $\lim_{x\to -4} \dfrac{x^2 - 3x - 28}{(x+4)^3}$

(f) $\lim_{x\to -\infty} \dfrac{(x^2 - 3x^3)^2(2x+1)}{(2x^2+1)^2(3x+4)^3}$

(g) $\lim_{x\to 2^+} \dfrac{|x-2|}{2x^3 - 4x^2 + 3x - 6}$

(2) **3.** Answer the following questions with True or False. Justify your answer.

(a) A function can have AT MOST two horizontal asymptotes.

(b) A function cannot intersect its vertical asymptote.

(3) **4.** Use the definition of continuity to determine the points of discontinuity of the following function, $f(x)$. Name the type of discontinuity.

$$f(x) = \begin{cases} \dfrac{3x+12}{x^2+2x-8} & : x \leq -2 \\ x^2 & : -2 < x < 2 \\ \dfrac{4}{3-x} & : x \geq 2 \end{cases}$$

(3) **5.** Find the value(s) of k for which the function $g(x)$ is continuous on $\mathbb{R}$.

$$g(x) = \begin{cases} k^2 + 2x & : x < -1 \\ -kx & : x \geq -1 \end{cases}$$

(3) **6.**

(a) State the limit definition of the derivative.

(b) Use this definition to calculate the derivative of $g(x) = \sqrt{3-2x}$

(21) **7.** Find the derivative of each of the following functions. Do not simplify your answers.

(a) $y = \frac{3}{2x^6} - \sqrt[3]{x^4} + \log_8 x + \pi$

(b) $g(x) = \cot(xe^x)$

(c) $f(x) = \sin^2(x^3 - 7^x)$

(d) $y = \dfrac{e^{9x}+1}{\ln(6x^3 - 3x^5) - 3}$

(e) $y = \sqrt[3]{\sec(2x^3+4)+6}$

(f) $(x+3y)^2 = x^2 + e^{2y}$

(g) $y = (\cos x + 4x)^{\sqrt{x}}$

(3) **8.** Use logarithmic differentiation to find the derivative of $f(x) = \dfrac{5(2^x - 6x^3)^5 \ln x}{7(\tan x)\sqrt[4]{(3x^2+2x+1)^5}}$

(4) **9.** Given $\dfrac{-4x^2}{y} = x \ln y - y$

(a) Find y'

(b) Find the equation of the tangent line at the point $(x, y) = (\frac{1}{2}, 1)$

(3) **10.** Find both y' and y'' for $y^2 = xy + 8$

(4) **11.** Find the value(s) where the tangent line to $g(x) = (x^2 + 4)^3(x^2 - 10)^4$ is horizontal.

(3) **12.** Find all critical numbers of the function $h(x) = x\sqrt{1 + x}$.

(3) **13.** Find the absolute extrema of the function $g(x) = \dfrac{x^3 + 4}{x^2}$ on the interval $[1, 4]$

(10) **14.** Consider

$$f(x) = \frac{x}{(2x+1)^2}, f'(x) = \frac{-2x+1}{(2x+1)^3}, \text{ and } f''`(x) = \frac{8(x-1)}{(2x+1)^4}$$

Determine the following then neatly sketch a graph of $f(x)$ on the following page.

(a) all $x-$ and $y-$ intercepts

(b) all vertical and horizontal asymptotes

(c) the intervals on which $f(x)$ is increasing and decreasing

(d) all local (relative) maxima and minima

(e) the intervals on which $f(x)$ is concave up and concave down

(f) any points of inflection

(g) sketch the curve $y = f(x)$ on the following page

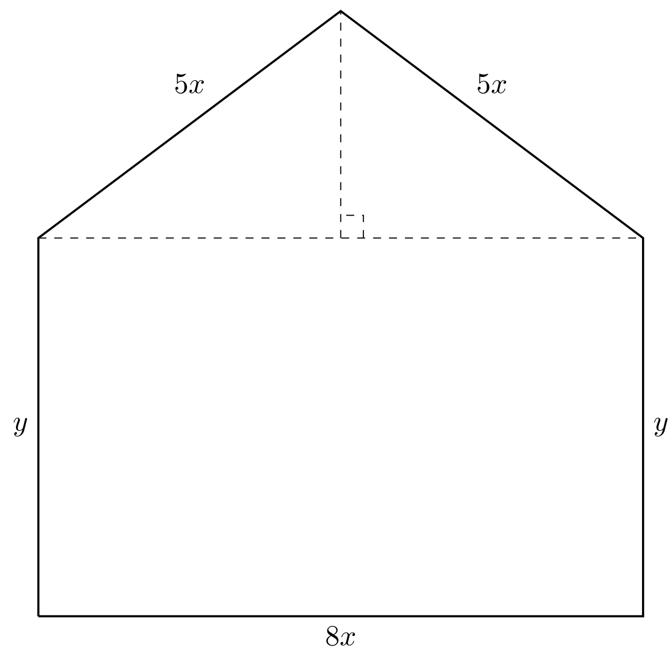
(3) **15.** Canadian Apparel prints orange T-shirts featuring the basic differentiation rules. It costs $2000 to set up the printer and $12 to produce and print a T-shirt, that is, the cost function of producing x T-shirts is $C(x) = 2000 + 12x$.

(a) Find average cost function.

(b) What happens to the average cost as the number of T-shirts gets large (that is, as $x \to \infty$)?

(c) Find the marginal average cost at the production level of 300 T-shirts and interpret the result.

(4) **16.** A group of sadistic calculus teachers decided to lock up some adorable unicorns in an unusual room, because reasons. The room is a rectangle with a isosceles triangle on top, where the base of the rectangle (and triangle) is $8x$ and the sides of the triangle are $5x$.



What is the maximum area for the room, if the total length of the walls of the room add up to 2640m?

(4) **17.** The demand for a mini samurai robot unicorn is given by $p^2 + 4x = 72081$.

(a) Find the elasticity of demand function, η, in terms of p.

(b) By calculating the value of η when the price is $p = \$9$ (and $x = 18000$), determine what will happen to quantity demanded if the price is increased by 6%.

(c) Does the revenue increase or decrease if the price increase in (b) is approved?

Answers

1. a)-1 b)1 c)-3 d)2 e)-2 f)$-\infty$ g)2 h)dne i)dne
   j)-3 : jump, -2 : removable, 2 : infinite
   k)$-1, 1$: corner, abrupt change in slope

2. a)$\frac{2}{49}$ b)$\frac{-1}{4\sqrt{12}} = \frac{-1}{8\sqrt{3}}$ c)0 d)$\frac{5}{6}$ e)$-\infty$ f)$\frac{1}{6}$ g)$\frac{1}{11}$

3. a)True. There can be one on the left as x approaches $-\infty$ and one on the right at x approaches ∞.
   b) False. It is possible for there to be exactly one point ON the vertical asymptote, but the function cannot intersect it.

4. Removable discontinuity at $x = -4$, infinite discontinuity at $x = 3$, and a jump discontinuity at $x = -2$.

5. $k = -1, 2$

6. (a) $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

   (b) $f'(x) = \dfrac{-1}{\sqrt{3-2x}}$

7. (a) $y' = -9x^{-5} - \frac{4}{3}x^{1/3} + \frac{1}{x\ln 8}$
   (b) $g'(x) = -\csc^2(xe^x)(e^x + xe^x)$
   (c) $f'(x) = 2\sin(x^3 - 7^x)\cos(x^3 - 7^x)(3x^2 - 7^x \ln 7)$
   (d) $y' = \dfrac{(9e^{9x})[\ln(6x^3 - 3x^5) - 3] - (e^{9x} + 1)\left(\frac{18x^2 - 15x^4}{6x^3 - 3x^5}\right)}{[\ln(6x^3 - 3x^5) - 3]^2}$
   (e) $y' = \frac{1}{3}[\sec(2x^3 + 4) + 6]^{-2/3}\sec(2x^3 + 4)\tan(2x^3 + 4)(6x^2)$
   (f) $y' = \dfrac{2x - 2(x+3y)}{6(x+3y) - 2e^{2y}} = \dfrac{-3y}{3x + 9y - e^{2y}}$
   (g) $y' = (\cos x + 4x)^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\ln(\cos x + 4x) + \sqrt{x}\left(\frac{-\sin x + 4}{\cos x + 4x}\right)\right)$

8. $f'(x) = \left(\dfrac{5(2^x - 6x^3)^5 \ln x}{7(\tan x)\sqrt[4]{(3x^2 + 2x + 1)^5}}\right)\left(\dfrac{5(2^x \ln 2 - 18x^2)}{2^x - 6x^3} + \dfrac{1}{x\ln x} - \dfrac{\sec^2 x}{\tan x} - \dfrac{5(6x+2)}{4(3x^2 + 2x + 1)}\right)$

9. a) $y' = \dfrac{y^2 \ln y + 8xy}{4x^2 - xy + y^2}$ b) $y = \frac{8}{3}x - \frac{1}{3}$

10. $y' = \dfrac{y}{2y - x}$ and $y'' = \dfrac{-2xy + 2y^2}{(2y - x)^3}$

11. The tangent line is horizontal with $x = \pm\sqrt{10}, x = \pm\sqrt{2}$, and $x = 0$.

12. $x = \frac{-2}{3}$

13. $g(x)$ has an absolute minimum at $(2, 3)$ and an absolute maximum at $(1, 5)$

14. (a) Both x- and y- intercept at $(0, 0)$.
    (b) Vertical asymptote at $x = -\frac{1}{2}$, horizontal asymptote at $y = 0$: $y \to 0$ as $x \to \pm\infty$
    (c) Increasing on $(-\frac{1}{2}, \frac{1}{2})$ and decreasing on $(-\infty, -\frac{1}{2}) \cup (\frac{1}{2}, \infty)$
    (d) Local max at $(\frac{1}{2}, \frac{1}{8})$
    (e) Concave down on $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{2})$ and concave up on $(\frac{1}{2}, \infty)$
    (f) Inflection point at $(1, \frac{1}{9})$.

15. (a) $\overline{C(x)} = \frac{2000}{x} + 12$
    (b) $\lim_{x\to\infty} \overline{C(x)} = 12$, so average cost approaches \$12
    (c) Marginal average cost at 300 is $\approx$ \$0.02

16. Maximum area is $316,800m^2$ and is achieved when $x = 120m$ and $y = 240m$.