1. Evaluate the following integrals.

(a) $\displaystyle\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$

(b) $\displaystyle\int \frac{x\arcsin(x^2)}{\sqrt{1-x^4}}\,dx$

(c) $\displaystyle\int \frac{x+6}{x(x^2+2x+3)}\,dx$

(d) $\displaystyle\int \sin(\ln x)\,dx$

(e) $\displaystyle\int \frac{1}{x^3\sqrt{x^2-4}}\,dx$

(f) $\displaystyle\int \sqrt{\frac{3+x}{3-x}}\,dx$

2. Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\sin(2x))}$

(b) $\displaystyle\lim_{x\to \pi/2^-} (\tan x)^{2x-\pi}$

(c) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{3}{e^{3x}-1}\right)$

3. Evaluate each improper integral or show that it diverges.

(a) $\displaystyle\int_0^\infty (-xe^{-x})\,dx$

(b) $\displaystyle\int_0^2 \frac{1}{(x-1)^{2/3}}\,dx$

4. Give the solution of the differential equation $\cos x\,\dfrac{dy}{dx} = \sin x\sqrt{y^2+4}$ which satisfies $y=0$ if $x=0$.

5. Find the area of the region bounded by $y_1 = x^3+x^2+3x+1$ and $y_2 = x^3+x+4$.

6. Let $\mathcal{R}$ be the region bounded by $x=0$, $f(x)=1+x$ and $g(x)=x^3+x$. Set up, **but do not evaluate**, an integral which represents the volume obtained by revolving $\mathcal{R}$ about:

(a) the x-axis;

(b) the line $x=3$.

7. Find the arc length function for the curve $x = \frac{1}{4}y^2 - \frac{1}{2}\ln y$, taking $\left(\frac{1}{4}, 1\right)$ as the starting point.

8. Determine with justification, whether the sequence $\{a_n\}$ converges or diverges. If a sequence converges, find its limit.

(a) $a_n = \left(\dfrac{3n+1}{3n-1}\right)^n$

(b) $a_n = \dfrac{n^3(2n)!}{(2n+2)!}$

9. For the telescoping series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right)$,

(a) give a formula for s_n, the sum of the first n terms of the series, and (b) find the sum of the series.

10. Determine whether each series is convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{\sqrt{n^2+3}}{3n^2+7}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$

11. Determine whether each series is absolutely convergent, conditionally convergent or divergent. Justify your answers by displaying proper solutions.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n!}{1 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot (2n+1)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{3^{n+1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{5n+3}}$

12. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{5^n \sqrt{n}}.$$

13. For the function $f(x) = \dfrac{1}{2+x}$, find the Taylor series around $x = 1$. Write the first four terms of the series explicitly, and express the series using appropriate sigma notation.