1. (a) $\int (8x+12)\sqrt{x^2+3x+9}\ \mathrm{d}x$

(b) $\int (x^3+2x)\cos(2x)\ \mathrm{d}x$

(c) $\int \frac{x^2+3x-14}{(x+1)(x+5)^2}\ \mathrm{d}x$

(d) $\int_2^7 2-|x-5|\ \mathrm{d}x$

(e) $\int \frac{t^2+t^{3/2}-5t^3\cot(t)}{t^3}\ \mathrm{d}t$

(f) $\int \frac{\cos(x)(\sin^2(x)+\tan(x))}{\sin(x)}\ \mathrm{d}x$

(g) $\int \frac{\sqrt{x}}{\sqrt{x}+3}\ \mathrm{d}x$

2. Find $f(x)$ given $f''(x) = \frac{3-3x}{2\sqrt{x}}$, $f'(1) = 4$, and $f(4) = 14$.

3. If the marginal cost of producing x units of dragon figurines is given by $\frac{dC}{dx} = 0.04x + 0.5$, and if producing 40 units costs \$13, find the *average* cost per unit of producing 60 units.

4. Find the area of the region bounded by the graphs of $f(x) = x^3 - 6x^2 + 9x$ and $g(x) = x^2 - 3x$.

5. Given the demand function $p_1(x) = \frac{12}{x+3}$ and the supply function $p_2(x) = x + 2$ of a unicorn figurine company,

(a) Find the equilibrium point.

(b) Sketch and identify the regions representing consumer and producer surplus.

(c) Evaluate Consumer Surplus.

6. Use Simpson's Rule to estimate the value of $\int_1^4 \sqrt{x^2+2}\ \mathrm{d}x$. Use $n = 4$, and round your answer to 4 decimal places.

7. Determine whether $y = xe^{2x}$ is a solution to the differential equation $y'' - 2y' = 2y$.

8. Solve the following differential equations.

(a) $y' = \frac{2x+\sec^2 x}{2y}$; $y(0) = 5$

(b) $\frac{dy}{dx} = y(x+2)$; $y(0) = 2$

9. In a city whose population is $200,000$ there is an outbreak of a flesh-eating zombie virus. When the city health department begins its record keeping, there are 225 zombies. The number of zombies N is increasing at a rate proportional to the square root number of zombies at time t in weeks. One week later, there are 625 zombies.

(a) Write a differential equation representing the problem.

(b) Find the function $N(t)$ for the number of zombies after t weeks.

(c) Find the number of zombies two weeks after the record keeping begins.

10. Evaluate the following limits:

(a) $\lim_{x\to\infty} \frac{\ln(x^2-7)}{x^2+4}$

(b) $\lim_{x\to 0} \frac{x-\sin(x)}{x^3-2x^2}$

11. Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

(a) $\int_3^5 \frac{x}{\sqrt{25-x^2}}dx$

(b) $\int_0^\infty \frac{e^x}{e^x+1}dx$

12. Consider the sequence given by $\left\{\frac{-2}{5}, \frac{1}{-15}, \frac{6}{45}, \frac{13}{-135}, \dots\right\}$

(a) Find the sixth term a_6.

(b) Find the general term a_n.

13. Determine if the following sequences converge or diverge. If the sequence converges, find its limit.

(a) $a_n = \frac{(-1)^n(1-n)}{n^2+3}$

(b) $a_n = \frac{(n+1)!}{n^2(n-1)!}$

14. Genji wants to save for 10 years in order to buy a \$30,000 ancestral samurai sword for his daughter Zelda. If he can invest at an interest rate of 2.4% compounded monthly, what amount should he deposit every month?

15. Determine if the following series converge or diverge. If the series converges, find its sum (when possible).

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

(b) $\sum_{n=1}^{\infty} \frac{3^n}{3^n+n}$

(c) $\sum_{n=1}^{\infty} \frac{2^{-n}+2^n}{3^n}$

(d) $\sum_{n=1}^{\infty} \frac{(n+1)!3^n}{n!4^n}$

Answers

1. (a) $\frac{8}{3}(x^2+3x+9)^{3/2}+c$
   (b) $\frac{1}{2}(x^3+2x)\sin(2x)+\frac{1}{4}(3x^2+2)\cos(2x)-\frac{3}{4}x\sin(2x)+\frac{3}{8}\cos(2x)+c$
   (c) $-\ln|x+1|+2\ln|x+5|-\frac{1}{x+5}+c$
   (d) 7/2
   (e) $\ln|t|-\frac{2}{\sqrt{t}}-5\ln|\sin(t)|+c$
   (f) $\frac{1}{2}\sin^2 x+x+c$
   (g) $x-6\sqrt{x}+18\ln|\sqrt{x}+3|+c$ OR $(\sqrt{x}+3)^2-12(\sqrt{x}+3)+18\ln|\sqrt{x}+3|+c$
2. $f(x)=2x^{3/2}-\frac{2}{5}x^{5/2}+2x+\frac{14}{5}$
3. $\overline{C(x)}=0.02x+0.5-\frac{39}{x}$ so $\overline{C(60)}=\$1.05$
4. $A=\displaystyle\int_0^3 x^3-7x^2+12x\,\mathrm{d}x+\int_3^4 -x^3+7x^2-12x=\frac{71}{6}$
5. (a) (1,3)
   (b) graph
   (c) Consumer Surplus= 0.45
6. 8.7242
7. No, it is not a solution
8. (a) $y=\sqrt{x^2+\tan(x)+25}$
   (b) $y=2e^{1/2x^2+2x}$
9. (a) $\dfrac{dN}{dt}=k\sqrt{N}$
   (b) $N(t)=(\frac{k}{2}t+c)^2=(10t+15)^2$
   (c) 1225 zombies
10. (a) 0
    (b) 0
11. (a) converges to 4
    (b) diverges
12. (a) converges to 0
    (b) converges to 1
13. (a) $a_6=\dfrac{-33}{(5)(3)^5}$
    (b) $a_n=\dfrac{(-1)^{n+1}(n^2-3)}{5(3)^{n-1}}$
14. \$221.01 once a month
15. (a) converges to $\frac{3}{4}$
    (b) diverges by TFD (Test for Divergence)
    (c) converges to $\frac{11}{5}$
    (d) converges by the ratio test