1. (30 points) Evaluate the following integrals.

(a) $\displaystyle\int \frac{x^2 - x + 13}{(x+1)(x^2+4)}\,dx$

(b) $\displaystyle\int e^{-2x}\sin(3x)\,dx$

(c) $\displaystyle\int_3^4 \frac{\tan^3(\pi/x)}{x^2}\,dx$

(d) $\displaystyle\int \frac{1}{x^4\sqrt{x^2-1}}\,dx$

(e) $\displaystyle\int \frac{1}{x\sqrt{2x-9}}\,dx$

(f) $\displaystyle\int x^5 e^{-x^2}\,dx$

2. (8 points) Evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{\arctan(2x) - \pi/2}{\sin(1/x)}$

(b) $\displaystyle\lim_{x\to 0} \left(1+\sin^2 x\right)^{4/x^2}$

3. (8 points) Evaluate each of the following improper integrals.

(a) $\displaystyle\int_{-1}^0 \frac{e^{1/x}}{x^2}\,dx$

(b) $\displaystyle\int_{e^2}^\infty \frac{\ln x}{(x\ln x - x)^2}\,dx$

4. (4 points) Find the length of the curve $y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$ on the interval $[1, e^2]$.

5. (9 points) Let $\Re$ be the region bounded by $y = \sqrt{x}$ and $y = \frac{1}{8}x^2$.
Set up, **but do not evaluate**, the integral needed to find:

(a) The area of $\Re$.

(b) The volume of the solid of revolution obtained by rotating $\Re$ about:

(i) The y-axis.

(ii) The x-axis.

(iii) The line $x = -2$.

6. (4 points) Find an explicit solution of the differential equation.

$$(xy^2 + x) + (x^2y - y)\frac{dy}{dx} = 0 \quad , \quad y(0) = -2$$

7. (4 points) A tank initially contains 100 L of water in which 25 g of salt has been dissolved. Pure water enters the tank at a rate of 5 L/min. The solution is kept thoroughly mixed and drains from the tank at rate of 5 L/min. How much salt is in the tank after 20 minutes?

8. (3 points) Given the sequence $\left\{ \frac{2}{1}, \frac{4}{2}, \frac{8}{6}, \frac{16}{24}, \frac{32}{120}, \frac{64}{720}, \ldots \right\}$

(a) Find a formula for the general term a_n.

(b) Determine whether the sequence is convergent or divergent.

9. (9 points) Determine whether the following series converge or diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{3/n^2}}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4n-1}{25n+1} \right)^{n/2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n+1} \cos(3/n)$

10. (8 points) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(2n)^{2n}}{(2n)!}$

11. (4 points) Find the radius and interval of convergence of the power series.

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x+2)^n}{5^n \sqrt{n+1}}$$

12. (4 points) Let $f(x) = \dfrac{1}{(1-2x)^2}$.

(a) Find the first 5 terms of the Maclaurin series of $f(x)$.

(b) Express the Maclaurin series of $f(x)$ in summation notation.

13. (5 points) (a) Given that $\displaystyle\int f(x)\,dx = x \arccos(5x) - \frac{1}{5}\sqrt{1-25x^2} + C$, find $f(x)$.

(b) Show (without actually calculating it) that the area under the curve $y = e^{\sqrt{x}}$ on the interval $[0, 1]$ is the same as the area under the curve $y = e^{\sin x} \sin(2x)$ on the interval $[0, \frac{\pi}{2}]$.

(c) Answer True or False (briefly justify).

(i) If $a_n > 0$ and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then $\displaystyle\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

(ii) If $a_n > 0$ and $\displaystyle\lim_{n\to\infty} n^2 a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

Answers

1.(a) $3\ln|x+1| - \ln(x^2+4) + \frac{1}{2}\arctan(\frac{x}{2}) + C$ (b) $\frac{-1}{13}(3e^{-2x}\cos(3x) + 2e^{-2x}\sin 3x) + C$

(c) $\frac{2-\ln 2}{2\pi}$ (d) $\frac{\sqrt{x^2-1}}{x} - \frac{1}{3}\left(\frac{\sqrt{x^2-1}}{x}\right)^3 + C$ (e) $\frac{2}{3}\arctan\left(\frac{\sqrt{2x-9}}{3}\right) + C$

(f) $\frac{1}{2}[-x^4e^{-x^2} - 2(x^2e^{-x^2} + e^{-x^2})] + C$ 2.(a) $-\frac{1}{2}$ (b) e^4 3.(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ 4. $\frac{e^4}{2}$

5.(a) $\int_0^4 \sqrt{x} - \frac{1}{8}x^2\,dx$ (b)(i) $2\pi\int_0^4 x(\sqrt{x} - \frac{1}{8}x^2)\,dx$ (ii) $\pi\int_0^4 \left(\sqrt{x}\right)^2 - \left(\frac{1}{8}x^2\right)^2\,dx$

(iii) $2\pi\int_0^4 (2+x)\left(\sqrt{x} - \frac{1}{8}x^2\right)\,dx$ 6. $y = -\sqrt{\frac{5}{1-x^2} - 1}$ 7. $\frac{25}{e}$ 8.(a) $a_n = \frac{2^n}{n!}$

(b) Conv. to 0 9.(a) Div. by comp. with $\sum\frac{1}{n}$ (b) Conv. by Root Test (c) Div. by Div. Test

10.(a) Cond. conv. (b) Div. 11. $R = 5$, IC: $-7 < x \leqslant 3$ 12. $\sum_{n=0}^{\infty}(n+1)2^nx^n$

13. (a)$\arccos(5x)$

(b) Hint: Show subs. $u = \sqrt{x}$ for $\int_0^1 e^{\sqrt{x}}\,dx$ gives the same as subs. $u = \sin x$ for $\int_0^{\pi/2} e^{\sin x}\sin(2x)\,dx$

(c)(i) False (ii)True