Question 1: (*27 pts*) Evaluate each of the following integrals.

a) $\displaystyle\int \frac{2x^5 - 7x^3e^{2x} - x^2}{3x^3}\,dx$ **b)** $\displaystyle\int_1^{e^2} \frac{\big(\ln(x)+1\big)^2}{3x}\,dx$ **c)** $\displaystyle\int \frac{\sin(x)\cos(x)}{\sqrt[3]{\sin^2(x)+7}}\,dx$ **d)** $\displaystyle\int (2x^2-1)e^{4x}\,dx$

e) $\displaystyle\int \frac{\cot(2x) - 4x\cos(2x) + 3\sec(2x)}{\cos(2x)}\,dx$ **f)** $\displaystyle\int_{-2}^{1} \Big(2 - |x+1|\Big)\,dx$ **g)** $\displaystyle\int \frac{x^4+4x^3-4x^2+1}{x^3-x^2}\,dx$

h) $\displaystyle\int_1^e 2x\,\ln(x)\,dx$

Question 2: (*4 pts*) Find $f(x)$, given that $f''(x) = \dfrac{1}{2x^{3/2}} + \dfrac{3}{4\sqrt{x}}$, with $f(0) = 3$ and $f(4) = -9$.

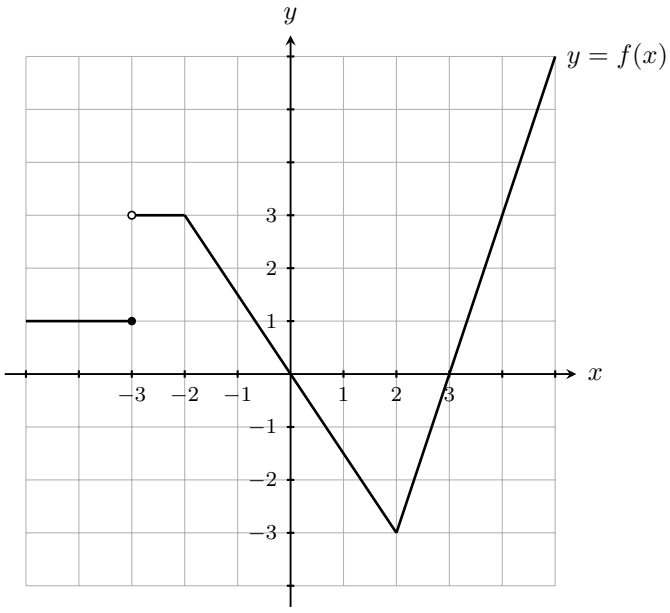
Question 3: (*4 pts*) Given the functions $f(x) = x^2 - 3$ and $g(x) = -2x$:

a) Find the intersection points of $f(x)$ and $g(x)$.
b) Find the area of the region between the graphs of f and g, between $x = -1$ and $x = 2$.

Question 4: (*4 pts*) Use Simpson's Rule (with $n = 6$) to approximate $\displaystyle\int_2^5 \frac{3}{\sqrt{4x^2+1}}\,dx$.

Round your answer to 4 decimal places.

Question 5: (*4 pts*) Use the graph of f below to evaluate the definite integrals:



a) $\displaystyle\int_{-5}^{-2} f(x)\,dx =$

b) $\displaystyle\int_{-2}^{3} f(x)\,dx =$

c) $\displaystyle\int_{-3}^{-3} f(x)\,dx =$

d) $\displaystyle\int_{3}^{0} f(x)\,dx =$

Question 6: (*5 pts*) Given the demand function $p = \sqrt{169 - 44x}$ and the supply function $p = x + 7$:

a) Find the equilibrium point.
b) Sketch and identify the regions representing the consumer and producer surpluses.
c) Evaluate the consumer surplus.

Question 7: (*4 pts*) Solve the differential equation $4yy'\sqrt{x^2+1} = 3x$, with $y(0) = 7$.

Question 8: (*6 pts*) A fund raiser is organized, with the objective of accumulating \$8000. Let A represent the amount raised after t hours. It was found that the rate of change of the amount collected is proportional to the difference between the amount raised and the goal. Of course, $A = \$0$ when $t = 0$, and it will take 5 hours to raise \$4000.

a) Write the differential equation for the problem.
b) Find the function $A(t)$ for the amount raised after t hours.
c) How long will it take to raise \$7500?

Question 9: (*6 pts*) Evaluate the following limits:

a) $\lim\limits_{x\to 0} \dfrac{x\sin(x)}{\cos^2(x) - 1}$ **b)** $\lim\limits_{x\to+\infty} \dfrac{4x+5}{\ln\left(3 + 2e^{7x}\right)}$

Question 10: (*8 pts*) Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

a) $\displaystyle\int_1^3 \frac{x+1}{\left(x^2+2x-3\right)^2}\,dx$ **b)** $\displaystyle\int_4^{+\infty} \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx$

Question 11: (*4 pts*) Determine if the following sequences converge or diverge. If the sequence converges, find its limit.

a) $a_n = \dfrac{(n^2+1)(2n+1)!}{(2n+3)!}$ **b)** $a_n = \dfrac{(-1)^{n+1}(n+2)}{5^n+4}$

Question 12: (*3 pts*) Consider the sequence $\left\{\dfrac{-3}{4}, \dfrac{7}{7}, \dfrac{-11}{12}, \dfrac{15}{19}, \cdots\right\}$

a) Give the 6^{th} term of the sequence. **b)** Find an expression for the general term of the sequence.

Question 13: (*4 pts*) A deposit of \$160.00 is made every 3 months, in a savings account that earns 5% annual interest, compounded quarterly. What will be the balance in the account after 20 years?

Question 14: (*17 pts*) Determine if the following series converge or diverge. Identify which test you are using. If the series converges, find its sum (when possible).

a) $\displaystyle\sum_{n=1}^{+\infty} \frac{3^{n+1}+2^n}{4^n}$ **b)** $\displaystyle\sum_{n=2}^{+\infty} \frac{4}{n^2-1}$ **c)** $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt{n}}{n^{1.3}}$ **d)** $\displaystyle\sum_{n=1}^{+\infty} \frac{\sqrt{n^4+3}}{3n^2+7}$ **e)** $\displaystyle\sum_{n=0}^{+\infty} \frac{(n+2)5^n}{7^{n+1}}$

ANSWERS

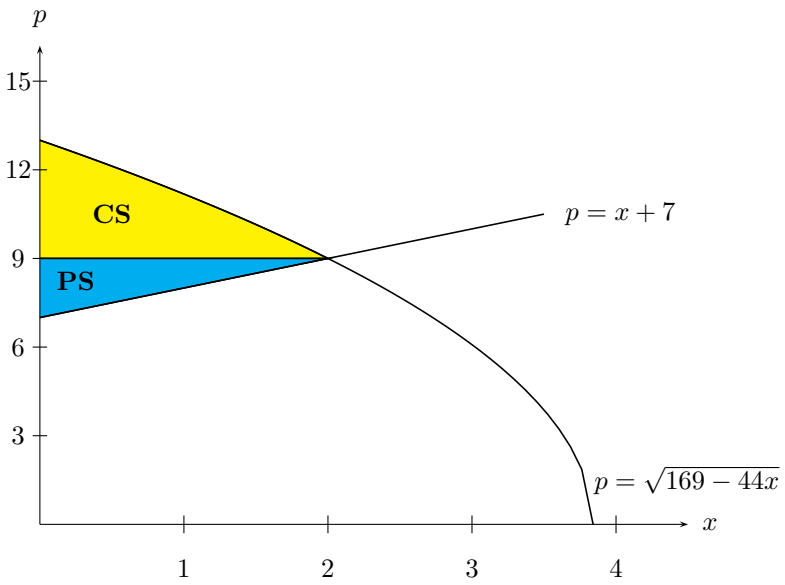
1.) a) $\frac{2}{9}x^3-\frac{7}{6}e^{2x}-\frac{1}{3}\ln|x|+C$ b) $\frac{26}{9}$ c) $\frac{3}{4}\left(\sin^2(x)+7\right)^{2/3}+C$ d) $\frac{1}{4}(2x^2-1)e^{4x}-\frac{1}{4}xe^{4x}+\frac{1}{16}e^{4x}+C$

e) $\frac{1}{2}\ln\left|\csc(2x)-\cot(2x)\right|-2x^2+\frac{3}{2}\tan(2x)+C$ f) $\frac{7}{2}$ g) $\frac{1}{2}x^2+5x-\ln|x|+\frac{1}{x}+2\ln|x-1|+C$

h) $\frac{e^2+1}{2}$ **2.)** $f(x)=-2x^{1/2}+x^{3/2}-4x+3$ **3.)** a) $(1,-2)$ and $(-3,6)$ b) $\frac{23}{3}$ **4.)** 1.3554

5.) a) 5 b) $\frac{-3}{2}$ c) 0 d) $\frac{9}{2}$ **6.)** a) E=$(2,9)$

b)



c) \$4.24 **7.)** $y=\sqrt{\frac{3}{2}\sqrt{x^2+1}+\frac{95}{2}}$ **8.)** a) $\frac{dA}{dt}=k(8000-A)$ b) $A=8000\left(1-2^{-t/5}\right)$

c) 20 hours **9.)** a) -1 b) $\frac{4}{7}$ **10.)** a) diverges b) $\frac{2}{e^2}$ **11.)** a) Converges to $\frac{1}{4}$

b) Converges to 0 **12.)** a) $\frac{23}{39}$ b) $\frac{(-1)^n(4n-1)}{n^2+3}$ **13.)** \$22 051.24

14.) a) Geometric series, converges to 10 b) Telescoping series, converges to 3

c) p-series, diverges d) Divergence test, diverges e) Ratio test, converges