

**Question 1: (27 pts)** Evaluate each of the following integrals.

$$\text{a) } \int \frac{2x^5 - 7x^3 e^{2x} - x^2}{3x^3} dx \quad \text{b) } \int_1^{e^2} \frac{(\ln(x) + 1)^2}{3x} dx \quad \text{c) } \int \frac{\sin(x) \cos(x)}{\sqrt[3]{\sin^2(x) + 7}} dx \quad \text{d) } \int (2x^2 - 1)e^{4x} dx$$

$$\text{e) } \int \frac{\cot(2x) - 4x \cos(2x) + 3 \sec(2x)}{\cos(2x)} dx \quad \text{f) } \int_{-2}^1 (2 - |x + 1|) dx \quad \text{g) } \int \frac{x^4 + 4x^3 - 4x^2 + 1}{x^3 - x^2} dx$$

$$\text{h) } \int_1^e 2x \ln(x) dx$$

**Question 2: (4 pts)** Find  $f(x)$ , given that  $f''(x) = \frac{1}{2x^{3/2}} + \frac{3}{4\sqrt{x}}$ , with  $f(0) = 3$  and  $f(4) = -9$ .

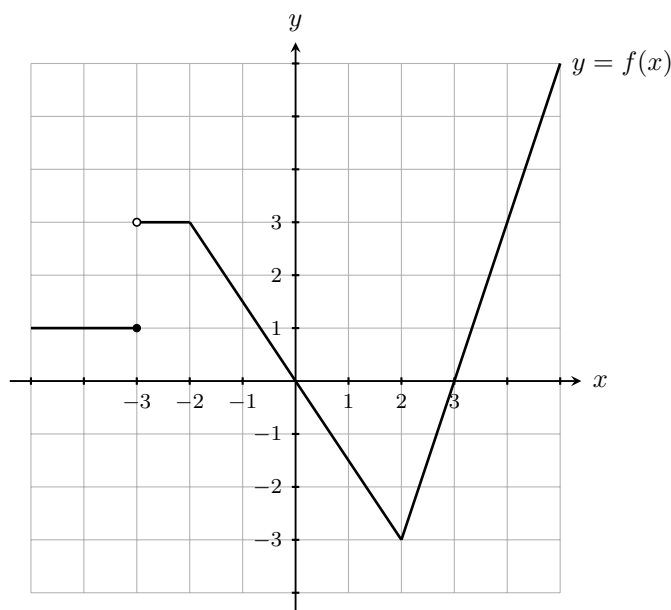
**Question 3: (4 pts)** Given the functions  $f(x) = x^2 - 3$  and  $g(x) = -2x$ :

- Find the intersection points of  $f(x)$  and  $g(x)$ .
- Find the area of the region between the graphs of  $f$  and  $g$ , between  $x = -1$  and  $x = 2$ .

**Question 4: (4 pts)** Use Simpson's Rule (with  $n = 6$ ) to approximate  $\int_2^5 \frac{3}{\sqrt{4x^2 + 1}} dx$ .

Round your answer to 4 decimal places.

**Question 5: (4 pts)** Use the graph of  $f$  below to evaluate the definite integrals:



$$\text{a) } \int_{-5}^{-2} f(x) dx =$$

$$\text{b) } \int_{-2}^3 f(x) dx =$$

$$\text{c) } \int_{-3}^{-2} f(x) dx =$$

$$\text{d) } \int_3^0 f(x) dx =$$

**Question 6: (5 pts)** Given the demand function  $p = \sqrt{169 - 44x}$  and the supply function  $p = x + 7$ :

- Find the equilibrium point.
- Sketch and identify the regions representing the consumer and producer surpluses.
- Evaluate the consumer surplus.

**Question 7: (4 pts)** Solve the differential equation  $4yy'\sqrt{x^2 + 1} = 3x$ , with  $y(0) = 7$ .

**Question 8: (6 pts)** A fund raiser is organized, with the objective of accumulating \$8000. Let  $A$  represent the amount raised after  $t$  hours. It was found that the rate of change of the amount collected is proportional to the difference between the amount raised and the goal. Of course,  $A = \$0$  when  $t = 0$ , and it will take 5 hours to raise \$4000.

- Write the differential equation for the problem.
- Find the function  $A(t)$  for the amount raised after  $t$  hours.
- How long will it take to raise \$7500?

**Question 9: (6 pts)** Evaluate the following limits:

$$\text{a) } \lim_{x \rightarrow 0} \frac{x \sin(x)}{\cos^2(x) - 1} \qquad \text{b) } \lim_{x \rightarrow +\infty} \frac{4x + 5}{\ln(3 + 2e^{7x})}$$

**Question 10: (8 pts)** Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

$$\text{a) } \int_1^3 \frac{x+1}{(x^2+2x-3)^2} dx \qquad \text{b) } \int_4^{+\infty} \frac{1}{\sqrt{x} e^{\sqrt{x}}} dx$$

**Question 11: (4 pts)** Determine if the following sequences converge or diverge. If the sequence converges, find its limit.

$$\text{a) } a_n = \frac{(n^2 + 1)(2n + 1)!}{(2n + 3)!} \qquad \text{b) } a_n = \frac{(-1)^{n+1}(n + 2)}{5^n + 4}$$

**Question 12: (3 pts)** Consider the sequence  $\left\{ \frac{-3}{4}, \frac{7}{7}, \frac{-11}{12}, \frac{15}{19}, \dots \right\}$

- Give the 6<sup>th</sup> term of the sequence.
- Find an expression for the general term of the sequence.

**Question 13: (4 pts)** A deposit of \$160.00 is made every 3 months, in a savings account that earns 5% annual interest, compounded quarterly. What will be the balance in the account after 20 years?

**Question 14: (17 pts)** Determine if the following series converge or diverge. Identify which test you are using. If the series converges, find its sum (when possible).

$$\text{a) } \sum_{n=1}^{+\infty} \frac{3^{n+1} + 2^n}{4^n} \quad \text{b) } \sum_{n=2}^{+\infty} \frac{4}{n^2 - 1} \quad \text{c) } \sum_{n=1}^{+\infty} \frac{\sqrt{n}}{n^{1.3}} \quad \text{d) } \sum_{n=1}^{+\infty} \frac{\sqrt{n^4 + 3}}{3n^2 + 7} \quad \text{e) } \sum_{n=0}^{+\infty} \frac{(n + 2)5^n}{7^{n+1}}$$

**ANSWERS**

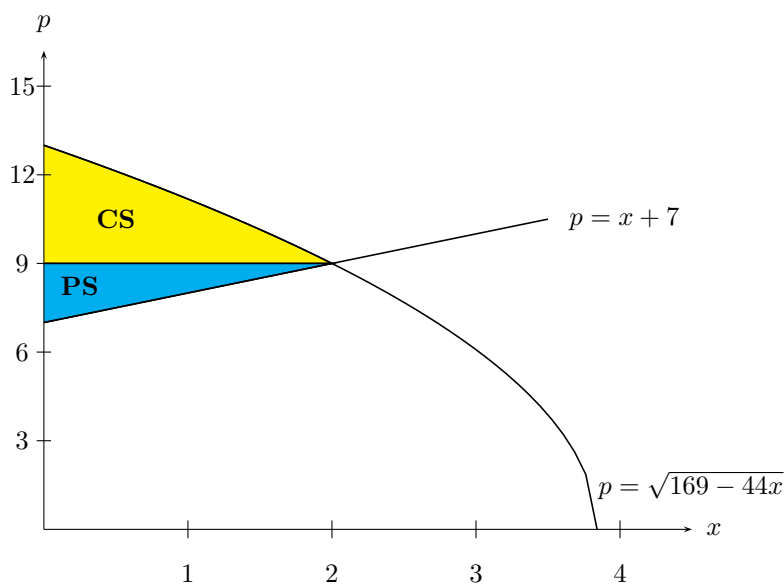
1.) a)  $\frac{2}{9}x^3 - \frac{7}{6}e^{2x} - \frac{1}{3}\ln|x| + C$  b)  $\frac{26}{9}$  c)  $\frac{3}{4}(\sin^2(x) + 7)^{2/3} + C$  d)  $\frac{1}{4}(2x^2 - 1)e^{4x} - \frac{1}{4}xe^{4x} + \frac{1}{16}e^{4x} + C$

e)  $\frac{1}{2}\ln|\csc(2x) - \cot(2x)| - 2x^2 + \frac{3}{2}\tan(2x) + C$  f)  $\frac{7}{2}$  g)  $\frac{1}{2}x^2 + 5x - \ln|x| + \frac{1}{x} + 2\ln|x-1| + C$

h)  $\frac{e^2 + 1}{2}$  2.)  $f(x) = -2x^{1/2} + x^{3/2} - 4x + 3$  3.) a) (1, -2) and (-3, 6) b)  $\frac{23}{3}$  4.) 1.3554

5.) a) 5 b)  $\frac{-3}{2}$  c) 0 d)  $\frac{9}{2}$  6.) a) E=(2, 9)

b)



c) \$4.24 7.)  $y = \sqrt{\frac{3}{2}\sqrt{x^2 + 1} + \frac{95}{2}}$  8.) a)  $\frac{dA}{dt} = k(8000 - A)$  b)  $A = 8000(1 - 2^{-t/5})$

c) 20 hours 9.) a) -1 b)  $\frac{4}{7}$  10.) a) diverges b)  $\frac{2}{e^2}$  11.) a) Converges to  $\frac{1}{4}$

b) Converges to 0 12.) a)  $\frac{23}{39}$  b)  $\frac{(-1)^n(4n-1)}{n^2+3}$  13.) \$22 051.24

14.) a) Geometric series, converges to 10 b) Telescoping series, converges to 3

c) p-series, diverges d) Divergence test, diverges e) Ratio test, converges