1. (5 points) Given the graph of f below, determine each of the following. Use ∞, $-\infty$ or "does not exist" where appropriate.

(a) $\lim_{x\to 1^-} f(x) =$

(b) $\lim_{x\to -2} f(x) =$

(c) $f(-2) =$

(d) $\lim_{x\to -\infty} f(x) =$

(e) $\lim_{x\to 2} f(x) =$

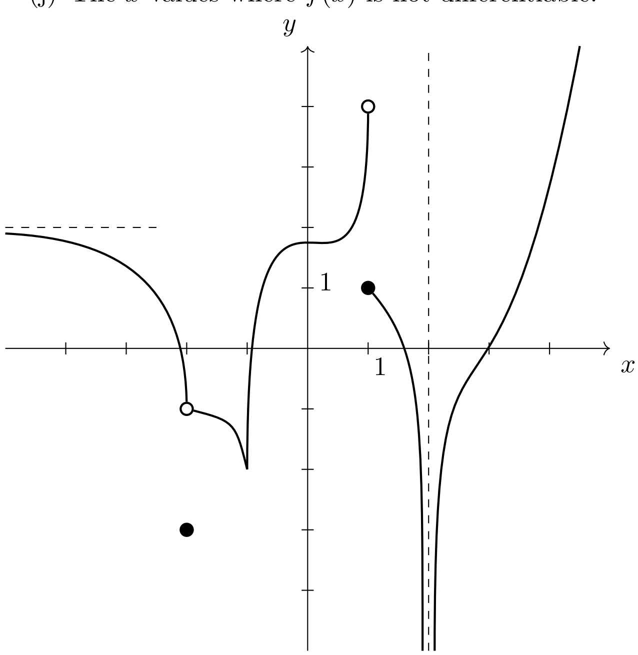
(f) $\lim_{x\to 1} f(x) =$

(g) $\lim_{x\to \infty} f(x) =$

(h) $\lim_{x\to 3^-} \dfrac{1}{f(x)} =$

(i) The x-values where $f(x)$ is not continuous:

(j) The x-values where $f(x)$ is not differentiable:



2. (21 points) Evaluate each of the following limits (3 marks each). If a limit does not exist, use "does not exist", ∞ or $-\infty$ as appropriate.

(a) $\lim_{x\to 1} \dfrac{2x^2-9x+7}{5x^2-7x+2}$

(b) $\lim_{x\to 10^+} \dfrac{x+5}{x+10}$

(c) $\lim_{x\to -1} \dfrac{\sqrt{10+6x}-2}{x^2-2x-3}$

(d) $\lim_{x\to 0} \dfrac{\dfrac{9}{3-x}-(x+3)}{x^2}$

(e) $\lim_{x\to -\infty} \dfrac{x(x-3)(x-5)}{2-x-x^2}$

(f) $\lim_{x\to 3^-} \dfrac{|x-3|}{x^2-3x}$

(g) $\lim_{x\to 1} \dfrac{2x^2-8x+14}{5x^2-7x-2}$

3. (3 points) By using the definition of continuity, find the points of discontinuity for the following function $f(x)$. Name the type of each discontinuity.

$$f(x) = \begin{cases} \dfrac{(x+1)(x-4)}{(x-2)(x+1)} & \text{if } x < 1 \\[2ex] \dfrac{3}{2-x} & \text{if } x \geq 1 \end{cases}$$

4. (3 points) Find the value(s) of k such that the following function is continuous for all real numbers x.

$$g(x) = \begin{cases} \dfrac{3}{x-k} & \text{if } x < 2 \\[2ex] \dfrac{k}{1-x} & \text{if } x \geq 2 \end{cases}$$

5. (4 points)

(a) State the limit definition of the derivative.

(b) Use this definition to find the derivative of $f(x) = \sqrt{5-3x}$.

6. (18 points) Find $\frac{dy}{dx}$ for each of the following (3 marks each). Do not simplify your answers.

(a) $y = 2x^e + \frac{4}{\sqrt{x}} + \sqrt[3]{x^5}$

(b) $y = \log_4\left(8x^3 - 6x\right) + 6^{4x^2-3x+1} + e^x \tan\left(3x^4\right)$

(c) $y = \sec^3\left(\cos\left(1-x\right)\right)$

(d) $\frac{x}{y} = \ln y - x^2 y$

(e) $y = \frac{\cot\left(3x+1\right) + 4x^3}{8x^4 - 4x^8}$

(f) $y = \left(3x + \sqrt{x}\right)^{x^3-2x}$

7. (3 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ with $y = \frac{6e^x \sqrt[5]{x^3+6x^2}}{(2x-9)^6 \ln x}$.

8. (5 points) Given $\left(x^3 + y^2 + 7\right)^4 = 4x + 3y + 9$,

(a) find y',

(b) find the equation of the tangent line at $(-2, 0)$.

9. (4 points) Find the fifth derivative $y^{(5)}$ for $y = \sin x + e^{3x+1} + x^4 + 8$.

10. (3 points) Given $f(x) = e^x(x^2 - 3)$ find the absolute extrema for the function on $[-1, 3]$.

11. (3 points) Use the second derivative test to find all local (relative) extrema of $f(x) = 18x^2 - 3x^3$.

12. (10 points) Let

$$f(x) = \frac{4}{x^2 - 4x}$$
$$f'(x) = \frac{8(2-x)}{\left(x^2-4x\right)^2}$$
$$f''(x) = \frac{8(3x^2 - 12x + 16)}{\left(x^2-4x\right)^3}$$

Determine the following characteristics then neatly sketch a graph of $f(x)$ on the following page including all the pertinent information.

(a) all x and y-intercepts,

(b) all vertical and horizontal asymptotes,

(c) all intervals on which $f(x)$ is increasing and decreasing,

(d) all local (relative) maxima and minima of $f(x)$,

(e) the intervals on which $f(x)$ is concave up and concave down,

(f) all points of inflection,

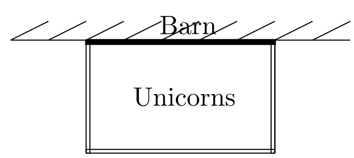
(g) sketch the curve $y = f(x)$ on the following page.

13. (4 points) A unicorn farm usually has very strict regulations about the number of guests that can be visiting at one time as too many guests disturbs the unicorns. Recently though they haven't been earning enough for the upkeep of the unicorns. The price per person is usually \$84, but to encourage guests they have started a promotion for big groups. For every guest over 10 they will decrease the price for everyone by \$2.

(a) How much should the unicorn farm owners charge for maximum revenue?

(b) How many people will visit when the revenue is maximized?

14. (5 points) A ranch owner is planning on building a new rectangular fence to enclose his field of unicorns. The long side against the barn needs to be made from steel, but the short sides and the long side facing the customers coming to see the unicorns should be made from rainbow. Steel costs \$60 per foot while rainbow costs \$80 per foot. If the area of the enclosure will be 89 600 square feet, what should the dimensions of the fence be in order to minimize the cost?



15. (4 points) The Rainbow Manufacturing Ltd. has established that the revenue function is $R(x) = 2x^3 + 40x^2 + 8x$ per foot of rainbow and the cost function in dollars is $C(x) = 3x^3 + 19x^2 + 80x - 800$.

(a) Find the marginal profit.

(b) Find the price per unit to maximize the profit.

16. (5 points) Assume that the demand equation of a product is $p = 42 + x - x^2$.

(a) Find the price elasticity of demand function η.

(b) Is the demand elastic or inelastic at $x = 6$?

(c) When the price is \$12 at $x = 6$, what will happen to the quantity demanded if the price increases by 2%?

Solutions

1.

(a) 4
(b) -1
(c) -3
(d) 2
(e) $-\infty$
(f) DNE
(g) ∞
(h) $-\infty$
(i) $x=-2$, $x=1$, $x=2$
(j) $x=-2$, $x=-1$, $x=1$, $x=2$

2.

(a) $-\frac{5}{3}$
(b) $\frac{3}{4}$
(c) $-\frac{3}{8}$
(d) $\frac{1}{3}$
(e) ∞
(f) $-\frac{1}{3}$
(g) -2

3. $x=-1$ removable discontinuity, $x=2$ infinite discontinuity

4. $k=3$ ($k=-1$ is rejected because it makes the denominator of the first fraction 0)

5.

(a) $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(b) $f'(x)=-\frac{3}{2\sqrt{5-3x}}$

6.

(a) $y'=2ex^{e-1}-\frac{1}{2}x^{-\frac{3}{2}}+\frac{5}{3}x^{\frac{2}{3}}$

(b) $y'=\frac{1}{\ln 4}\frac{24x^2-6}{8x^3-6x}+6^{4x^2-3x+1}\ln 6(8x-3)+e^x\tan(3x^4)+e^x\sec^2(3x^4)12x^3$

(c) $y'=3\sec^2(\cos(1-x))\sec(\cos(1-x))\tan(\cos(1-x))[-\sin(1-x)](-1)$

(d) $y'=\frac{2xy^3+y}{x+y-x^2y^2}$

(e) $y'=\frac{(-\csc^2(3x+1)\cdot 3+12x^2)(8x^4-4x^8)-(\cot(3x+1)+4x^3)(32x^3-32x^7)}{(8x^4-4x^8)^2}$

(f) $y'=\left[(3x^2-2)\ln(3x+\sqrt{x})+(x^3-2x)\frac{1}{3x+\sqrt{x}}\left(3+\frac{1}{2\sqrt{x}}\right)\right]y$

7. $y'=\left[1+\frac{1}{5}\frac{3x^2+12x}{x^3+6x^2}-\frac{12}{2x-9}-\frac{1}{\ln x}\cdot\frac{1}{x}\right]y$

8.

(a) $y'=\frac{4-12(x^3+y^2+7)^3x^2}{8(x^3+y^2+7)^3y-3}$

(b) $y=-\frac{52}{11}x-\frac{104}{3}$

9. $y^{(5)}=\cos x+e^{3x+1}\cdot 3^5+0$

10.
critical numbers: $x=1$, $x=-3$
absolute minimum at $x=1$ with $f(1)=-2e$
absolute maximum at $x=3$ with $f(3)=6e^3$

11.
critical numbers: $x=0$, $x=4$
local minimum at $x=0$ with $f(0)=0$
local maximum at $x=4$ with $f(4)=96$

12.

(a) no x-intercepts, no y-intercept

(b) vertical asymptotes: $x = 0$, $x = 4$
horizontal asymptotes: $y = 0$

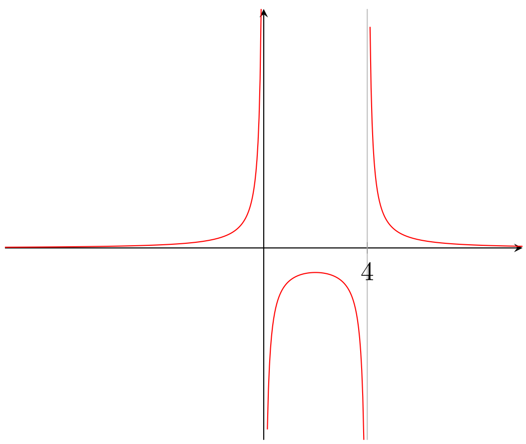
(c) $f(x)$ increases on $(-\infty, 0)$, $(0, 2)$
$f(x)$ decreases on $(2, 4)$, $(4, \infty)$

(d) no local minima
local maximum at $x = 2$ with $f(2) = -1$

(e) $f(x)$ is concave up on $(-\infty, 0)$ and $(4, \infty)$
$f(x)$ is concave down on $(0, 4)$

(f) no points of inflection

(g) the sketch of the curve:



13.

(a) maximal revenue: \$52

(b) optimal number of visitors: 26

14.
optimal sizes: long side 320 ft, short side 280 ft
minimal cost: \$ 89600

15.

(a) marginal profit: $P'(x) = -3x^2 + 42x - 72$

(b) optimal production level: $x = 12$, maximal profit: \$776

16.

(a) $\eta(x) = \dfrac{42 + x - x^2}{x - 2x^2}$

(b) $\eta(6) = -\dfrac{2}{11} > -1$, inelastic demand

(c) the demand will decrease approximately by 0.36%