[10] 1. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to -2} \frac{x^2+2x}{x^2+6x+8}$

(b) $\displaystyle\lim_{x\to -2^-} \frac{x+1}{4-x^2}$

(c) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{2x^2+1}}{3x-5}$

(d) $\displaystyle\lim_{x\to 4} \frac{\frac{1}{x}-\frac{1}{4}}{2-\sqrt{x}}$

(e) $\displaystyle\lim_{x\to 0} \frac{\tan x - \sin(2x)}{x}$

[4] 2. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x+1}{x^2+x} & \text{if } \quad x < -1 \\ ax+b & \text{if } \ -1 \le x < 2 \\ x^2-2 & \text{if } \quad x \ge 2 \end{cases}$$

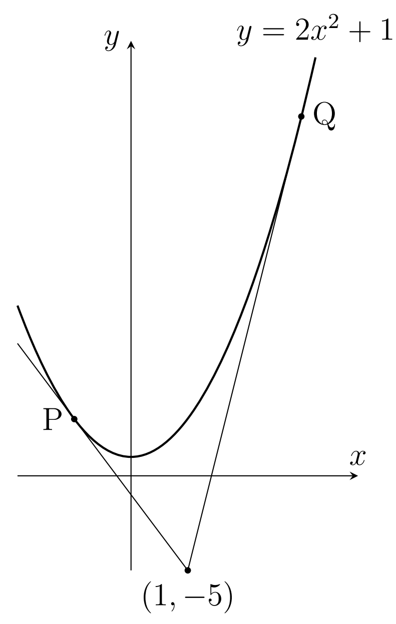
[3] 3. Sketch the graph of a function f such that all the following conditions are satisfied:

- $f(-5)=0$, $f(-\frac{1}{2})=0$ and $f(3)$ is undefined;
- $\displaystyle\lim_{x\to -4} f(x)=\infty$, $\displaystyle\lim_{x\to 1^-} f(x)=\infty$ and $\displaystyle\lim_{x\to 1^+} f(x)=-\infty$;
- $\displaystyle\lim_{x\to \infty} f(x)=-3$.

[4] 4. Find the derivative of $f(x)=\sqrt{x^2+1}$, using the limit definition of the derivative. Verify your answer using the derivative rules.

[1] 5. Evaluate $\displaystyle\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right)-1}{h}$. (Hint : Interpret this as a derivative.)

[3] 6. Find (both coordinates of) each point on the parabola defined by $y=2x^2+1$ at which the tangent line passes through the point $(1,-5)$.



[4] 7. Find an equation of the tangent line to the curve $x^2 + 2xy + 4y^2 = 13$ at the point $(-1, 2)$.

[15] 8. Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify your answers.

(a) $y = \cos^2(x)\sec(x^2) + \log_3 x + \pi^e$

(b) $y = \dfrac{\tan^2(e^x - 3)}{\ln(3x^2 + 5)}$

(c) $y = (\ln(\cos(e^{3x+7})))^6$

(d) $y = (\cot x)^{\sin x}$

(e) $y = \sqrt[4]{\dfrac{x^5 \sin^2 x}{(x-5)^6}}$ (Use logarithmic differentiation.)

[3] 9. Prove that the equation $x^3 + 33x - 8 = 0$ has exactly one root. Use the intermediate value theorem and Rolle's theorem in your proof.

[5] 10. Given the function $f(x) = \dfrac{2}{x^2} - \dfrac{9}{x^4}$

(a) state the equations of all horizontal and vertical asymptotes of f

(b) find the intervals on which f is increasing or decreasing

(c) find all local maximum or minimum values of f

[9] 11. Given

$$f(x) = x(x-5)^{2/3} \qquad f'(x) = \frac{5(x-3)}{3(x-5)^{1/3}} \qquad f''(x) = \frac{10(x-6)}{9(x-5)^{4/3}}$$

with $3(2^{2/3}) \approx 5$, find:

(a) the domain of f,

(b) x- and y- intercepts,

(c) vertical and horizontal asymptotes, if any,

(d) intervals on which f is increasing or decreasing,

(e) local extrema,

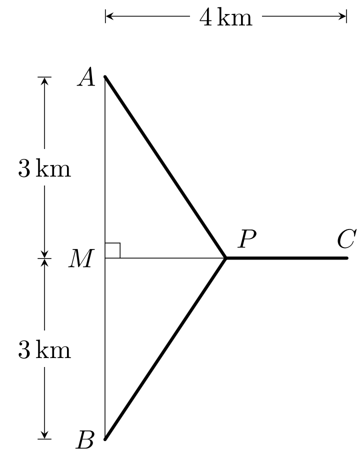
(f) intervals on which f is concave upward or downward,

(g) inflection point(s).

Sketch the graph of f. Label all intercepts, asymptotes, extrema and inflection point(s).

[4] 12. Find the absolute maximum and minimum of $f(t) = 4t^3 - 5t^2 - 8t + 3$ on $[-1, 1]$.

[6] 13. Factory A is 6 kilometres north of factory B, while power plant C is 4 kilometres east of the midpoint M of AB. Power is to be delivered to these two factories via a cable that will run from C to some point P (as in the diagram), where it will split into two branches going to A and B. How far away from the midpoint M should the branch point P be located in order to minimize the total length of the cable between A, B and C?



[3] 14. A particle moves in a straight line and has acceleration given by $a(t) = 6t+4$ cm/s^2. Its initial velocity is $v(0) = -6$ cm/s. and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

[4] 15. Compute $\int_0^2 (2x^3 - 1)dx$ as a limit of Riemann sums. Note that $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$.

[12] 16. Evaluate each of the following integrals.

(a) $\int \left(e^x + x^3 + 3^x + e^3\right) dx$

(b) $\int \frac{(2x+\sqrt{x})^2}{x^3} dx$

(c) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec\theta \tan\theta \csc\theta \, d\theta$

(d) $\int_{-3}^{2} |2x-1| \, dx$

[3] 17. Evaluate the following limit by expressing it as a definite integral.

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt[3]{\frac{1}{n}} + \sqrt[3]{\frac{2}{n}} + \sqrt[3]{\frac{3}{n}} + ... + \sqrt[3]{\frac{n}{n}}\right)$$

[3] 18. Use the Fundamental Theorem of Calculus to find the **second derivative** ($g''(x)$) of $g(x) = \int_{\ln(x)}^{x} te^t dt$.

[4] 19. **True** or **False**? Justify your answers!

(a) If $f(x) = \frac{x^3 - 4x}{x-2}$, then f has a vertical asymptote at $x = 2$.

(b) If f is continuous at $x = a$ then it must be differentiable at $x = a$.

(c) If $\int f(x)dx = x^2 \ln x + C$, then $f(x) = x + 2x\ln x$.

(d) $\int_{\pi}^{\pi} \sqrt{\tan x}\, dx = 0$