(Marks)

(7) 1. The number system called **Hexadecimal** uses as “digits” the 16 characters from the set $H = \{0, 1, 2, \dots, 9, a, b, c, d, e, f\}$. (In other words, every finite string of these symbols represents a number in Hexadecimal, and different strings represent different numbers.)

(a) How many distinct strings are possible that contain 5 digits if repetition is allowed?

(b) How many distinct strings are possible that contain 5 digits if repetition is not allowed?

(c) How many subsets of the 16 characters contain exactly 5 elements?

(d) How many proper subsets does H have?

(e) How many subsets of H contain at most 2 characters?

(5) 2. Let $U = \{0, 1, 2, \dots, 9, a, b, c, d, e, f\}$ be a universal set. Let $A = \{3, 7, 8, a, b, f\}$, $B = \{2, 3, 4, a, e, f\}$ and $C = \{x \mid x$ is also a digit in the decimal system.$\}$. List the elements of the following:

(a) $A \cup B$

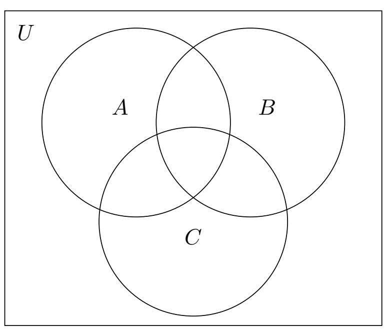
(b) $A \cup (B \cap C)$

(c) $A \cap \overline{C}$

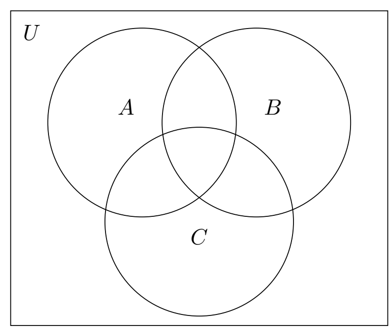
(d) $\overline{U \cup B}$

(e) Are A and B equivalent? Explain.

(4) 3. Use a Venn diagrams to illustrate the following property of sets: $C \cap (\overline{A \cap B}) = (C \cap \overline{A}) \cup (C \cap \overline{B})$. For each side of the equation, hatch two different sets in different ways. Then specify whether the final set is cross-hatched or anything hatched.



$C \cap (\overline{A \cap B})$



$(C \cap \overline{A}) \cup (C \cap \overline{B})$

(4) 4. Use truth tables to determine if the following is a tautology, a contradiction, or neither: $(p \wedge q)\veebar(p \barwedge q)$

(6) 5. Use truth tables to determine if the following statements are equivalent: $p \to (q \leftrightarrow r)$ and $(p \to q) \leftrightarrow (p \to r)$.

(3) 6. Given the following statements, fill in the blanks below with the words, converse, inverse, or contrapositive.

R: I’ll pass the course if I pass the exam.
S: I’ll pass the exam if I pass the course.
T: I won’t pass the course if I don’t pass the exam.

(a) Statement S is the ______________________________ of R.

(b) Statement T is the ______________________________ of S.

(c) Statement T is the ______________________________ of R.

(4) 7. Translate the following argument into symbolic logic and determine its validity. Be sure to state the interpretation of your simple statements.

H: If I lose this time, I win next time.
I don’t win next time.

C: I don’t lose this time.

(4) 8. Use Venn Diagrams to determine if the following argument is valid. Be sure to name and label your sets.

H: Some math courses are challenges.
Some challenges are rewarding.

C: Some math courses are rewarding.

(Marks)

(3) 9. Create a Boolean Table for the following expression: $\overline{AB + A}$.

(2) 10. Draw a network diagram that represents the following Boolean Expression: $(\overline{A} + B)(C + D(E + F))$

(6) 11. Simplify each boolean expression. State the properties that you are using at each step.

(a) $\overline{A}(A + B)(\overline{C} + A)$

(b) $AB\overline{C} + A\overline{B} + AC$

(3) 12. Use properties of boolean algebras to show $(A + B)(\overline{A} + \overline{B}) = A\overline{B} + \overline{A}B$.

(6) 13. Given the system $\begin{aligned} 2x - 2y &= -2 \\ 10x + 6y &= 30 \end{aligned}$

(a) Approximate the solution of the system by graphing each line.

(b) Solve the system algebraically by substitution or elimination.
(How accurate was your approximation in part (a)?)

(10) 14. Given: $A = \begin{bmatrix} 2 & -4 & 1 \\ -3 & 0 & 1 \\ 2 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 1 \\ 2 & 0 \\ -3 & 5 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 3 \\ 2 & -3 \end{bmatrix}$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

find each of the following, if possible. If an operation is not possible, say why.

(a) I_2B

(b) I_2C

(c) $D^2 + BC$

(d) $2B - C^T$

(e) $CB + A$

(6) 15. Given $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ find A^{-1} using row operations and verify your answer using the definition of inverse.

(4) 16. Given $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 0 \\ 4 & -2 & 2 \end{bmatrix}$ show that A^{-1} does not exist.

(5) 17. Given $A = \begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$,

(a) Write the matrix equation $AX = B$ as a system of two linear equations in two unknowns x and y.

(b) Find the inverse matrix of A.

(c) Use A^{-1} to solve system of linear equations from part (a).

(14) 18. Use matrices and row reduction to solve, if possible, the following systems. If there are infinitely many solutions express each variable in terms of free variable(s).

(a) $\begin{aligned} 2x + y + \quad &= 3 \\ x - y + 1z &= 5 \\ x + 2y + 1z &= -1 \end{aligned}$

(b) $\begin{aligned} x + 2y - z &= 4 \\ 2x + y \quad &= 3 \\ x - y + z &= 0 \end{aligned}$

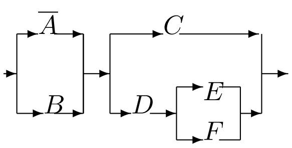
(c) $\begin{aligned} 3x - y + z &= 7 \\ 2x + y - z &= 3 \\ -8x - 4y + 4z &= -12 \end{aligned}$

(4) 19. Use mathematical induction to prove that the following statement is true for all positive integers n:

$$10 + 20 + 30 + \cdots + 10n = 5n(n + 1)$$

(Marks)

ANSWERS:

1. (a) $16^5 = 1048576$ (b) ${}_{16}P_5 = 524160$ (c) ${}_{16}C_5 = 4368$
   (d) $2^{16} - 1 = 65535$ (e) ${}_{16}C_0 +_{16} C_1 +_{16} C_2 = 137$
2. (a) $\{2, 3, 4, 7, 8, a, b, e, f\}$ (b) $\{2, 3, 4, 7, 8, a, b, f\}$ (c) $\{a, b, f\}$ (d) $\emptyset$
   (e) Yes, they have the same number of elements.
3.
4. It is a tautology, statement is true in every case.
5. Equivalent
6. (a) converse (b) contrapositive (c) inverse
7. Valid argument
8. Invalid argument
9. It is 0 when $A = 1$ and 1 when $A = 0$.
10. 

11. (a) $\overline{A}B\overline{C}$ (b) A
12. Use on the left side the distributive property followed by the complement property and then the identity property to get the right hand side.
13. (a) Lines intersect around $(1.5, 2.5)$ (b) $x = \frac{3}{2}, y = \frac{5}{2}$
14. (a) $\begin{bmatrix} -2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ (b) Undefined (c) $\begin{bmatrix} -7 & 2 \\ -2 & 38 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & -2 & 7 \\ 5 & 2 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} -3 & -3 & 13 \\ -7 & 0 & 5 \\ 23 & 5 & 12 \end{bmatrix}$
15. $\begin{bmatrix} 6 & -5 & -3 \\ -2 & 2 & 1 \\ -1 & 1 & 1 \end{bmatrix}$
16. No inverse, A can not be reduced to I.
17. (a) $\begin{array}{rcrcr} 2x & + & y & = & 8 \\ 7x & + & 5y & = & -3 \end{array}$ (b) $A^{-1} = \begin{bmatrix} 5/3 & -1/3 \\ -7/5 & 2/3 \end{bmatrix}$ (c) $X = A^{-1}B = \begin{bmatrix} 43/3 \\ -62/3 \end{bmatrix}$
18. (a) $\begin{array}{ll} x & = \frac{5}{2} \\ y & = -2 \\ z & = \frac{1}{2} \end{array}$ (b) No solution (c) $\begin{array}{ll} x & = 2 \\ y & = z - 1 \\ z & = z \end{array}$
19. $P_1 : 10 = 5(1)(2)$ true
    Assuming that P_k is true, P_{k+1} : LHS $= 5k(k + 1) + 10(k + 1) = 5k^2 + 15k + 10 = 5(k + 1)(k + 2) =$ RHS.