1. Evaluate each of the following integrals.

(3) (a) $\displaystyle\int \frac{x^{2/3} + 3x^2e^{3x} - 5 + 4x^2\csc^2(2x)}{x^2}\,dx$

(3) (b) $\displaystyle\int \frac{\sec^2(5x+4)}{1+2\tan(5x+4)}\,dx$

(3) (c) $\displaystyle\int 4x^3\ln x\,dx$

(3) (d) $\displaystyle\int 9x^2\sec(x^3+1)\,dx$

(3) (e) $\displaystyle\int_{-5}^{0} |x+4|\,dx$

(4) (f) $\displaystyle\int_{0}^{1} \frac{6x^3-5x^2-4x-2}{2x+1}\,dx$

(4) (g) $\displaystyle\int \frac{4x^2+7x+9}{(x+3)(x+1)^2}\,dx$

(4) (h) $\displaystyle\int (x+2)^2\,e^{3x}\,dx$

(4) 2. The marginal cost of a product is modelled by $\dfrac{dC}{dx} = \dfrac{1}{\sqrt{x}} + 5$. How much does it cost to produce 400 units if it costs \$ 145.00 to produce 100?

(4) 3. Use the Simpson's Rule with $n = 4$ to estimate $\displaystyle\int_1^9 \sqrt{x^3+1}\,dx$. Give the answer with 3 decimal places.

(4) 4. Find the function $f(x)$ that satisfies the conditions $f''(x) = 12x^2 - 6x$, $f'(-1) = 8$ and $f(-1) = 5$.

(4) 5. Find y given the differential equation $xy' = y(2-x)$ with condition $y(1) = 2$.

(4) 6. Given $f(x) = -x^2 + 4x$ and $g(x) = x^2 - 6$.

(a) Determine the points of intersection of $f(x)$ and $g(x)$.

(b) Determine the area of the region bounded by $f(x)$ and $g(x)$.

(6) 7. Given the demand function $p = -x^2 + 100$ and the supply function $p = 2x + 20$.

(a) Find the equilibrium point.

(b) Sketch and identify the regions representing the consumer and producer surpluses.

(c) Find the consumer surplus.

(d) Find the producer surplus.

(6) 8. Use l'Hôpital's rule to evaluate the following limits.

(a) $\lim\limits_{x \to 0} \dfrac{4x + 1 - e^{2x}}{4e^{3x} - 4}$

(b) $\lim\limits_{x \to \pi} \dfrac{\cos(\frac{x}{2})}{x - \pi}$

(8) 9. Evaluate each improper integral and state if it converges or diverges.

(a) $\displaystyle\int_0^1 \frac{x - 2}{\sqrt{x^2 - 4x + 3}}\, dx$

(b) $\displaystyle\int_1^\infty \frac{e^{3x}}{(3 - e^{3x})^2}\, dx$

(6) 10. Let V be the value of a stock at time t (in weeks). Assume that the maximum value of this stock is \$80.00. It was found that the rate of change in the value of the stock is proportional to the difference between its maximum value and its value V. Initially, the stock is valued at \$ 50.00 and after 3 weeks, the stock is worth \$56.00.

(a) Write the differential equation for the situation.

(b) Find an equation for $V(t)$.

(c) What will be the value of the stock after 6 weeks?

(4) 11. Determine if the following sequence converges or diverges. If the sequence converges, find its limit.

(a) $a_n = \dfrac{3^{n+2}}{7^n - 5}$

(b) $a_n = \dfrac{(n+4)!}{3n^2(n+1)!}$

(2) 12. Write the general term a_n of the sequence $\left\{ \dfrac{1}{2}, -\dfrac{2}{5}, \dfrac{6}{8}, -\dfrac{24}{11}, \ldots \right\}$

(4) 13. A deposit of \$200.00 is made every two months into a saving account that pays 3% interest a year, compounded every two months. Find the balance after 7 years.

(17) 14. Determine if the series converges or diverges. Identify the test you are using. Whenever possible, determine the sum of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2+5k+6}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{k^2+1}{\ln k}$

(d) $\displaystyle\sum_{n=1}^{\infty} n^{-4/5}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{3^{n+2}}$

Answers

1. (a) $-\dfrac{3}{x^{1/3}} + e^{3x} + \dfrac{5}{x} - 2\cot(2x) + C$ (b) $\frac{1}{10}\ln|1+2\tan(5x+4)| + C$ (c) $x^4 \ln x - \dfrac{x^4}{4} + C$

(d) $3\ln|\sec(x^3+1) + \tan(x^3+1)| + C$ (e) $\frac{17}{2}$ (f) $-1 - \ln 3$

(g) $6\ln|x+3| - 2\ln|x+1| - \dfrac{3}{x+1} + C$ (h) $\frac{1}{3}(x+2)^2 e^{3x} - \frac{2}{9}(x+2)e^{3x} + \frac{2}{27}e^{3x} + C$

2. $C(x) = 2\sqrt{x} + 5x - 375$; \$ 1,665.00 **3.** 97.492 **4.** $f(x) = x^4 - x^3 + 15x + 18$

5. $y = \dfrac{2x^2}{e^{x-1}}$ **6.** (a) $(-1,-5), (3,3)$ (b) $\frac{64}{3}$ squared units

7.(a) $(8, 36)$ (c) \$ 341.33 (d) \$ 64.00 **8.** (a) $\frac{1}{6}$ (b) $-\frac{1}{2}$

9. (a) $-\sqrt{3}$ (b) $\dfrac{1}{3(e^3-3)}$

10. (a) $\dfrac{dV}{dt} = k(80 - V)$ (b) $V(t) = 80 - 30\left(\dfrac{4}{5}\right)^{t/3}$ (c) \$ 60.80

11. (a) Converges to 0 (b) Diverges **12.** $\dfrac{(-1)^{n+1} n!}{3n-1}$ **13.** \$9,367.91

14. (a) Converges to $\dfrac{9}{8}$; Geometric Series (b) Converges to $\dfrac{1}{4}$; Telescoping Series

(c) Divergent; Divergence Test (d) Diverges; p-Series (e) Diverges; Ratio Test