



EPARTMENT OF ATHEMATICS
FINAL EXAMINATION

15 May 2015
14h-17h

MATHEMATICAL MODELS
201-225-AB

INSTRUCTOR: R.Masters

STUDENT NAME: _____

STUDENT NUMBER: _____

INSTRUCTOR: _____

INSTRUCTIONS

1. Do not open this booklet before the examination begins.
2. Check that this booklet contains 4 pages, excluding this cover page and the formula sheet.
3. Write all of your solutions in this booklet and show all supporting work.
4. If the space provided is not sufficient, continue the solution on the opposite page.

1. Find y' for the following. **Do not simplify your answer.**

(3) (a) $y = \sin(\arctan(3x)) + 7^{2x}$

(3) (b) $\arcsin(x + y) + 2y = x^3$ Hint: Solve for y' .

(3) (c) $y = \int_{\cos x}^1 (-2t + 3 \sin t) dt$ Hint: use part 1 of the F.T.C.

(3) (d) $y = \log_7(\tan x) + e^{\sec 5x}$

2. Given $f(x) = \frac{x}{x^2 - 9}$ $f'(x) = -\frac{(x^2 + 9)}{(x^2 - 9)^2}$ $f''(x) = \frac{(2x)(x^2 + 27)}{(x^2 - 9)^3}$.

Find (if any):

(1) (a) The x and y intercept(s).

(1) (b) The vertical and horizontal asymptotes.

(1) (c) The critical numbers

(1) (d) The inflection points.

(1) (e) Local (relative) extrema.

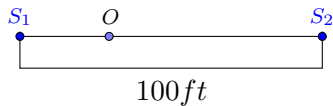
(1) (f) Intervals of upward or downward concavity.

(1) (g) Intervals on which f is increasing or decreasing.

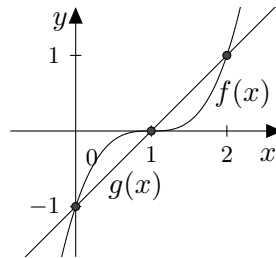
(3) (h) Sketch the graph of f .

(4) 3. Use Simpson's Rule with $n = 4$ to estimate the value of $\int_1^5 \frac{1}{x^2 + x} dx$ (give your answer to 3 decimals).

(5) 4. The illumination of an object (O) by a light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. If two light sources (S_1 and S_2), one 8 times as strong as the other, are placed 100 ft apart, how many feet from the brighter source should an object be placed on the line between the sources so as to receive the least illumination (see diagram below)?



(4) 5. Find the area of the region between the two graphs, $f(x) = (x - 1)^3$ and $g(x) = x - 1$.



6. Evaluate the following limits.

(3) (a) $\lim_{x \rightarrow \frac{\pi}{2}} (1 - \sin x) \cdot \tan x$

(3) (b) $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$

(4) (c) $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{\sin x}$

- (3) 7. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and the volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 600 cm^3 , the pressure is 150 kPa , and the pressure is increasing at a rate of 20 kPa/min . At what rate is the volume decreasing at this instant?

8. Solve the following differential equations for y .

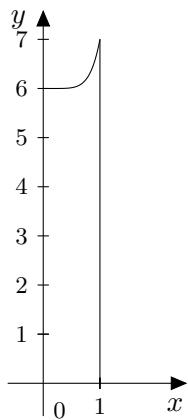
(3) (a) $ydt + tdy = 3ty dt$ with initial condition $y(1)=1$.

(3) (b) $(yx^2 + y)\frac{dy}{dx} = \tan^{-1} x$

- (4) 9. Find the volume of the solid formed by rotating the region enclosed by

$$x = 0, x = 1, y = 0, y = 6 + x^6$$

about the y -axis. Hint: Split the region into two.



- (3) 10. Determine if $y = x^3 + 2x^2 - 5$ is a solution to the differential equation $-y''' + y'' - \frac{y'}{x} = 3x - 6$

11. Integrate the following integrals.

(4) (a) $\int \frac{\cos(\ln x)}{x} dx$

(4) (b) $\int \tan^3 x dx$

(4) (c) $\int \frac{1}{\sqrt{x^2 + 6x + 13}} dx$

(4) (d) $\int \frac{5}{x^2 - 9} dx$

(4) (e) $\int \tan^9 x \sec^4 x dx$

(4) (f) $\int \arcsin x dx$

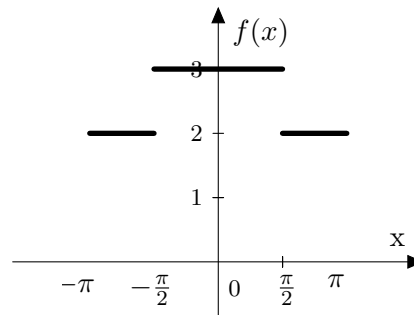
(4) (g) $\int \sin^2(2x) dx$

(5) 12. Solve the following first order linear differential equation for y.

$$\frac{dy}{dx} + 2y \cot x = 4 \cos x \quad \text{with initial condition } x = \pi/2 \text{ when } y = 1/3$$

13. Given the function and its graph.

$$f(x) = \begin{cases} 2, & \text{if } -\pi \leq x < -\frac{\pi}{2}, \frac{\pi}{2} \leq x < \pi \\ 3, & \text{if } -\frac{\pi}{2} \leq x < \frac{\pi}{2} \end{cases}$$



(2) (a) Determine if the function is even or odd. Show your work or Explain to obtain full marks

(4) (b) Find the first three non-zero terms of the Fourier series for the function above. Hint: find $a_0, a_1, a_2, a_3, \dots, b_1, b_2, \dots$ and write the function expansion.**Answers**

1. (a) $y' = \cos(\arctan 3x) \cdot \frac{3}{1 + (3x)^2} + 2 \cdot 7^{2x} \cdot \ln 7$

(b) $y' = \frac{3x^2 \sqrt{1 - (x+y)^2} - 1}{1 + 2\sqrt{1 - (x+y)^2}}$

(c) $y' = \frac{\sec^2 x}{\tan x \cdot \ln 7} + e^{\sec 5x} \cdot 5 \sec 5x \tan 5x$

(d) $y' = (2 \cos x - 3 \sin(\cos x))(-\sin x)$

2. $A \approx 0.522$ 3. (a) x -int = 0 and y -int = 0(b) V.A. $x = \pm 3$; H.A. $y = 0$

(c) None

(d) $x = 0$

(e) None

(f) C.U.] - 3, 0[\cup] 3, ∞ [C.D.] - ∞ , -3[\cup] 0, 3[

- (g) Inc. No where
dec.] $-\infty, \infty$ [
4. $\frac{200}{3}ft$ away from source 2
5. $A = 1/2$
6. (a) 0
(b) $1/2$
(c) 1
7. $\frac{dV}{dt} = -80 \text{ cm}^3/\text{min}$
8. (a) $y = \frac{e^{3(t-1)}}{t}$
(b) $y = \sqrt{(\arctan x)^2 + 2c}$
9. $\frac{25}{4}\pi$
10. yes
11. (a) $\sin(\ln x) + c$
(b) $\frac{\tan^2 x}{2} - \ln|\sec x| + c$
(c) $\ln \left| \frac{\sqrt{x^2 + 6x + 13} + x + 3}{2} \right| + c$
(d) $\ln \left| \frac{x-3}{x+3} \right|^{5/6} + c$
(e) $\frac{\tan^{12} x}{12} + \frac{\tan^{10} x}{10} + c$
(f) $x \cdot \arcsin x + \sqrt{1-x^2} + c$
(g) $1/2(x - 1/4 \sin x) + c$
12. $y = 4/3 \sin x - \csc^2 x$
13. (a) $f(x)$ is even
(b) $f(x) = \frac{5}{2} + \frac{2}{\pi} \cos x - \frac{2}{3\pi} \cos x + \frac{2}{5\pi} \cos x - \dots$