1. Evaluate each of the following integrals, without the use of integration table.

(3) (a) $\displaystyle\int \frac{x^3 - 2^x x^2 + (x-1)^2}{x^2}\,dx$

(3) (b) $\displaystyle\int_0^1 (t^2+1)e^{t^3+3t}\,dt$

(3) (c) $\displaystyle\int \frac{u+2}{2u+1}\,du$

(3) (d) $\displaystyle\int_1^e \frac{\ln(x)}{x^4}\,dx$

(4) (e) $\displaystyle\int \frac{4x}{(x^2-1)(x+1)}\,dx$

(3) (f) $\displaystyle\int \csc^2(x)\Big(\cot(x)-1\Big)^{3/2}\,dx$

(4) (g) $\displaystyle\int (x-8)^2\sin(2x)\,dx$

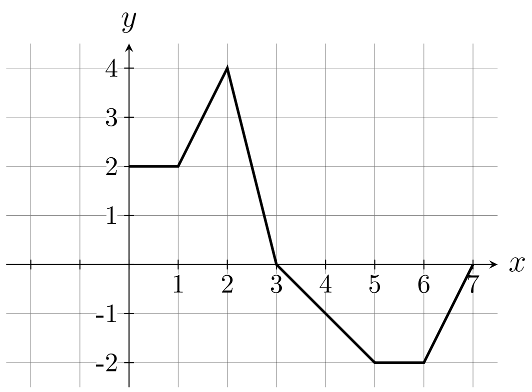
(3) (h) $\displaystyle\int \frac{\sec(x)\tan(x)-\cos(x)}{\cos(x)\tan(x)}\,dx$

(3) 2. The marginal revenue for the sale of a product is given by $\dfrac{dR}{dx} = 15 + \dfrac{2}{\sqrt{x}}$ where x is the quantity sold. Of course, no sale provides no revenue.

(a) Find the revenue function $R(x)$.

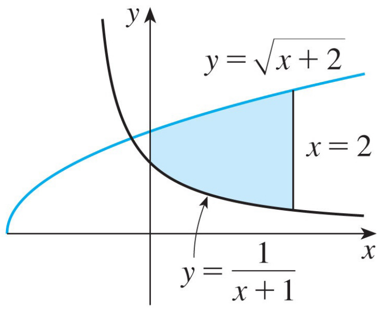
(b) What is the revenue when 100 items are sold?

(4) 3. Use the graph of $f(x)$ shown below to evaluate each of the following integrals:



(a) $\int_0^2 f(x)dx$ (b) $\int_2^0 f(x)dx$ (c) $\int_2^2 f(x)dx$ (d) $\int_2^5 f(x)dx$

(3) 4. Find the area of the shaded region.



(3) 5. The demand function for a certain product is given by $p = \dfrac{25}{\sqrt{x+16}}$, and the supply function by $p = \sqrt{x+16}$.

(a) Find the equilibrium point for the product.

(b) Find the producer surplus.

6. Use the integration table to find the following indefinite integral. In each case, state the formula number and justify its use.

(4) (a) $\displaystyle\int \frac{1}{(x+1)^2\sqrt{3-2x-x^2}}\,dx$

(4) (b) $\displaystyle\int \frac{x^5}{(3-4x^3)^2}\,dx$

7. Solve the differential equation for y.

(4) (a) $\dfrac{dy}{dx} = y(x+3)$ if $y = 7$ when $x = 0$ and $y > 0$.

(4) (b) $\dfrac{-y'}{y^2} = \sec(x)\tan(x)$ if $y = 1$ when $x = \pi$.

(5) 8. When an invasive species of beetle is colonizing a new area, its population increases at a rate inversely proportional to the population (P). The population began with 4 beetles (at $t = 0$ years), and after one year there were 1000 beetles.

(a) Write the differential equation for the situation.

(b) Find an equation for $P(t)$.

(c) What is the beetle's population after 5 years?

(6) 9. Use l'Hôpital's rule to evaluate the following limits.

(a) $\lim_{x\to 1} \dfrac{\ln(x) - x + 1}{x^3 - 9x^2 + 15x - 7}$

(b) $\lim_{x\to\infty} \dfrac{e^x + 4x - 3}{3x^2 - 7}$

(8) 10. Determine whether the improper integral diverges or converges. If the integral converges, find its value.

(a) $\displaystyle\int_1^e \frac{1}{x(\ln x)^{3/2}}\,dx$

(b) $\displaystyle\int_3^\infty \frac{x}{(x^2+1)^3}\,dx$

11. Determine if the following sequence converges or diverges. If the sequence converges, find its limit.

(3) (a) $a_n = \dfrac{5n^2 - 1}{4^n + 9}$

(3) (b) $a_n = \dfrac{7n^2(n+1)!}{(n+3)!}$

(3) 12. Write the general term a_n of the sequence $\left\{\dfrac{4}{5}, \dfrac{7}{10}, \dfrac{10}{20}, \dfrac{13}{40}, \dots\right\}$

(4) 13. Use the Trapezoidal rule with $n = 5$ to approximate $\displaystyle\int_1^6 \frac{x^3}{\ln(x^2+1)}dx$. Round your answer to 3 decimal places.

14. Determine with justification if the series converges or diverges. Find the sum if the series converges.

(3) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+7}{\sqrt{4n^2+1}}$

(3) (b) $\displaystyle\sum_{n=0}^{\infty} \left[\frac{4}{3^n} + \frac{2^{n+1}}{5^n}\right]$

(3) 15. Use a geometric series to express $7.1\overline{5}$ as a quotient of two integers.

(4) 16. Mr. Doyle plans to invest \$40 every month in an account for 15 years. If the account pays 1.8%, compounded every month, how much will he have at the end of the 15 years?

Answers

1. (a) $\frac{x^2}{2} - \frac{2^x}{\ln 2} + x - 2\ln|x| - \frac{1}{x} + C$ (b) $\frac{e^4-1}{3}$ (c) $\frac{u}{2} + \frac{3}{4}\ln|2u+1| + C$

(d) $\frac{e^3-4}{9e^3}$ (e) $\ln|x-1| - \ln|x+1| - \frac{2}{x+1} + C$ (f) $-\frac{2}{5}\Big(\cot(x)-1\Big)^{5/2} + C$

(g) $-\frac{(x-8)^2}{2}\cos(2x) + \frac{(x-8)}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$ (h) $\tan(x) - \ln|\sin(x)| + C$

2. (a) $R(x) = 15x + 4\sqrt{x}$ (b) \$1,540

3. (a) 5 (b) -5 (c) 0 (d) 0

4. $\frac{16 - 3\ln 3 - 4\sqrt{2}}{3}$ squared units

5. (a) $(x_0, p_0) = (9, 5)$ (b) \$4.33

6. (a) $-\frac{\sqrt{3-2x-x^2}}{4(x+1)} + C$ (b) $\frac{1}{48}\Big(\ln|3-4x^3| + \frac{3}{3-4x^3}\Big) + C$

7. (a) $y = 7e^{\frac{x^2}{2}+3x}$ (b) $y = \frac{1}{2+\sec(x)}$

8. (a) $\frac{dP}{dt} = k\frac{1}{P}$ (b) $P(t) = \sqrt{999984t+16}$ (c) 2236

9. (a) $\frac{1}{12}$ (b) ∞

10. (a) Diverges (b) Converges to $\frac{1}{400}$

11. (a) Converges to 0 (b) Converges to 7

12. $a_n = \frac{3n+1}{5(2)^{n-1}}$

13. 108.282

14. (a) Diverges by the divergence test. (b) Sum of geometric series. Converges to $\frac{28}{3}$.

15. $\frac{322}{45}$

16. \$8,271.04