Total points: 100

(2) 1. Simplify the following expressions, expressing the answers with positive exponents only. You can assume that all variables are positive.

$$\left(\frac{27x^{-1}y^{-3}z^{2}}{3y^{-1}z^{-3}}\right)^{-2}$$

Solution:

$$\frac{x^2y^4}{81z^{10}}$$

(2) 2. Simplify the following expression:

$$3\sqrt{80} - 7\sqrt{45} + 2\sqrt{20}$$

Solution:

$$3\sqrt{80} - 7\sqrt{45} + 2\sqrt{20} = 12\sqrt{5} - 21\sqrt{5} + 4\sqrt{5} = -5\sqrt{5}$$

(2) 3. Rationalize the numerator of $\frac{\sqrt{4-x}+\sqrt{x+4}}{8x}$ and simplify.

Solution:

$$-\frac{1}{4(\sqrt{4-x} - \sqrt{x+4})}$$

(6) 4. Factor the following polynomials completely.

(a) $2x^3 - 12x^2 + 18x$

Solution:

$$2x(x-3)^2$$

(b) $x^2y^3 - y^3 + 64x^2 - 64$

Solution:

$$(x-1)(x+1)(y+4)(y^2-4y+16)$$

(c) $12x^2 - 14x - 40$

Solution:

$$2(3x+4)(2x-5)$$

(3) 5. Divide and simplify $\dfrac{x^2+5x-6}{2xy-2y} \div \dfrac{36-x^2}{6xy-36y}$

Solution:

$$-3$$

(4) 6. Add and simplify $\dfrac{4x+1}{x-8} - \dfrac{3x+2}{x+4} - \dfrac{49x+4}{x^2-4x-32}$.

Solution:

$$\frac{x-2}{x+4}$$

(3) 7. Simplify the following complex fraction: $\dfrac{\frac{2x+18}{x+1}}{\frac{2}{x+1} - \frac{3}{x-3}}$.

Solution:

$$-2x+6 \text{ or } -2(x-3)$$

(3) 8. Use long division to find the quotient and the remainder of $\dfrac{12x^4 - 9x^3 + 7x^2 + 9x - 15}{3x^2+4}$

Solution:

$$\begin{array}{r|rrrrr}
 & & & 4x^2 & -3x & -3 \\ \hline
3x^2+4 & 12x^4 & -9x^3 & +7x^2 & +9x & -15 \\
 & -12x^4 & & -16x^2 & & \\ \hline
 & & -9x^3 & -9x^2 & +9x & \\
 & & 9x^3 & & +12x & \\ \hline
 & & & -9x^2 & +21x & -15 \\
 & & & 9x^2 & & +12 \\ \hline
 & & & & 21x & -3
\end{array}$$

(4) 9. Consider the two points $A(-2,4)$ and $B(3,2)$. Find

(a) The equation of the vertical line through A.

Solution:

$$x = -2$$

(b) An equation of the line passing through A and B.

Solution:

$$y = -\frac{2}{5}x + \frac{16}{5}$$

(c) The midpoint of the line segment $\overline{AB}$

Solution:

$$\left(\frac{1}{2}, 3\right)$$

(d) The distance between A and B.

Solution:

$$\sqrt{29}$$

(1) 10. Find k such that the line through the points $A(-4, 2)$ and $B(3, -5)$ is parallel to the line through $C(1, 5)$ and $D(k, 2)$.

Solution: $k = 4$.

(5) 11. Consider the line L given by $x - 4y = 8$.

(a) Find the x-intercept and the y-intercept of L.

Solution: The x-intercept is 8 and the y-intercept is -2.

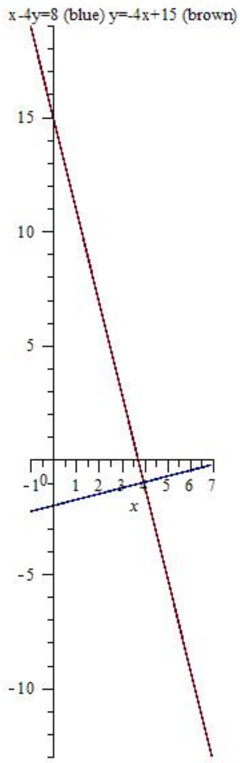
(b) Find the equation of the line P through the point $A(4, -1)$ and perpendicular to L.

Solution:

$$y = -4x + 15$$

(c) Graph both lines in the same coordinate system.

Solution:



12. Solve for x or state that there is no solution. Give exact answers.

(1) (a) $3x > 5x - 9$

Solution:

$$x < \frac{9}{2}$$

(2) (b) $x^2 = 7x - 12$

Solution:

$$x = 4, 3$$

(3) (c) $x^4 - 3x^2 - 10 = 0$

Solution:

$$x = \pm\sqrt{5}$$

(3) (d) $\dfrac{3}{2(x-4)} + \dfrac{2x+3}{2(x+4)} = \dfrac{12}{x^2-16}$

Solution:

$$x = -3$$

(3) (e) $\sqrt{x+3} + \sqrt{x+8} = 5$

Solution:

$$x = 1$$

(2) (f) $4^{x+3} = 2^{3x}$

Solution:

$$x = 6$$

(2) (g) $\log_2(x) + \log_2(x-6) = 4$

Solution:

$$x = 8$$

(1) (h) $2(e^x) = 7$

Solution:

$$x = \ln \frac{7}{2}$$

(2) 13. Find the domain of the function. Express your answer in interval notations.

$$f(x) = \frac{\sqrt{2x+6}}{x-6}$$

Solution:

$$[-3, 6) \cup (6, +\infty)$$

(2) 14. Let $f(x) = \frac{3x-4}{x^2+1}$ and $g(x) = \sqrt{2x+3}$

(a) Find $f(g(x))$ (do not simplify).

Solution:

$$f(g(x)) = \frac{3\sqrt{2x+3} - 4}{\left(\sqrt{2x+3}\right)^2 + 1}$$

(b) Find $g \circ g(x)$ (do not simplify).

Solution:

$$g \circ g(x) = \sqrt{2\sqrt{2x+3}+3}$$

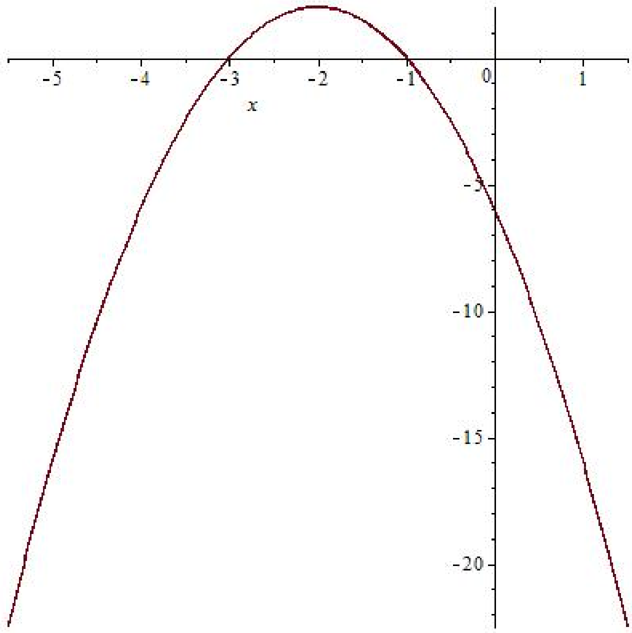
(3) 15. Let $f(x) = -2x^2 - 8x - 6$.

(a) Find the x- and y-intercept, the coordinates of the vertex and the axis of symmetry of $f(x)$.

Solution: x-intercept: -3 and -1; y-intercept: -6; the vertex $(-2, 2)$; the axis of symmetry: $x = -2$.

(b) Sketch the graph of $f(x)$.

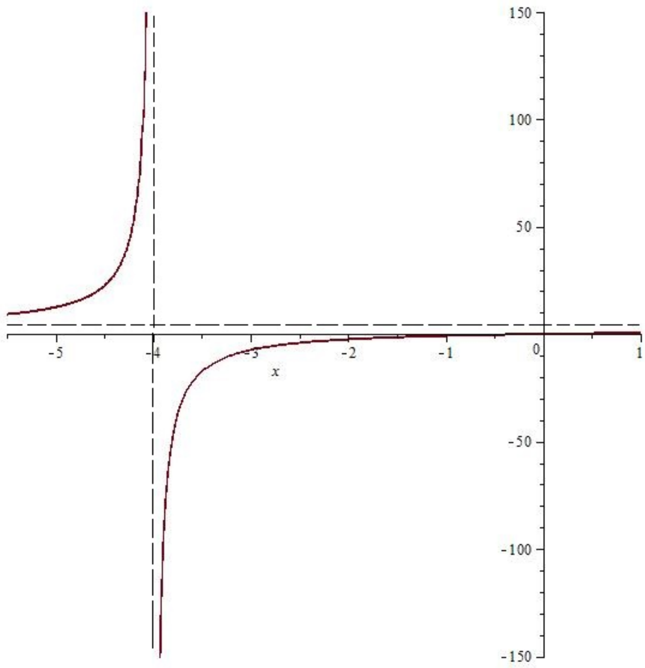
Solution:



(6) 16. Let $f(x) = \dfrac{5x}{2x+8}$.

(a) Sketch the graph of $f(x)$ (indicate its intercepts and asymptotes).

Solution:
x- and y-intercept are both $(0, 0)$. Vertical asymptote is $x = -4$ and horizontal asymptote is $y = \frac{5}{2}$.



(b) Find the inverse $f^{-1}(x)$ of the function $f(x)$.

Solution:

$$y = \frac{-8x}{2x-5}$$

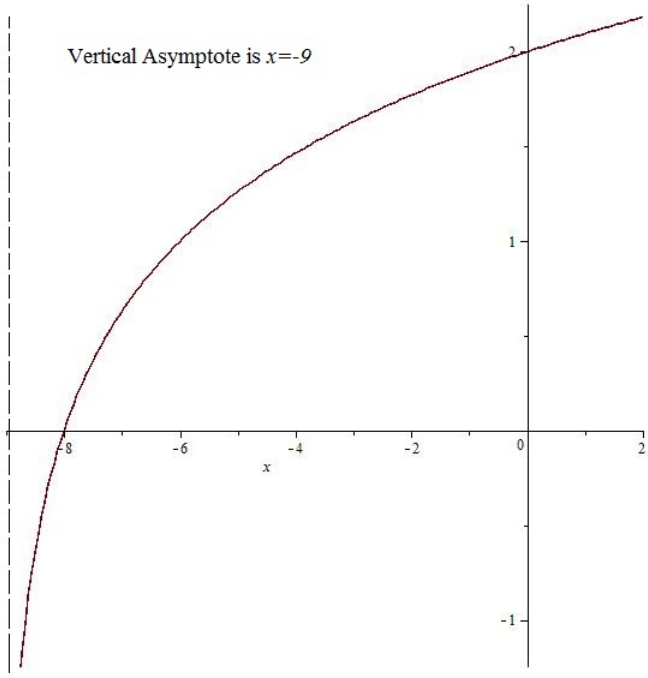
(1) 17. What would be the value of $5000 invested at annual interest rate 6% compounded quarterly after 20 years.

Solution:

$16453.31

(3) 18. Sketch the graph of $y = \log_3(x+9)$ (Indicate the intercepts and asymptotes).

Solution:



(2) 19. If $\ln\left(\frac{(2x+1)\sqrt{2x+1}}{\sqrt[3]{2x-1}}\right) = A\ln(2x+1) + B\ln(2x-1)$, what are A and B?

Solution:

$$A = \frac{3}{2},\ B = -\frac{1}{3}.$$

(2) 20. Express as a single logarithm: $\frac{1}{4}\ln x - 5\ln y + 3\ln y^2$.

Solution:

$$\ln\left(\frac{x^{\frac{1}{4}}y^6}{y^5}\right) = \ln\left(x^{\frac{1}{4}}y\right)$$

(2) 21. If $\cos\theta = 3/7$ and θ is acute, find $\sin\theta$, $\csc\theta$ and $\cot\theta$ (give **exact** values).

Solution:

$$\sin\theta = \frac{\sqrt{40}}{7},\ \csc\theta = \frac{7}{\sqrt{40}},\ \cot\theta = \frac{3}{\sqrt{40}}$$

(1) 22. Convert $5\pi/12$ to degrees.

Solution:

$$75^\circ$$

(1) 23. Convert $-390°$ to radians.

Solution:

$$-\frac{13\pi}{6}$$

(3) 24. A hot air balloon is secured to the ground with two ropes, one on each side. One rope makes an angle of 45° with the ground. The other rope makes an angle of 35° with the ground. If the two locations on the ground where the ropes are tied are 30m apart, how high is the balloon? (Give 3 decimal places.)

Solution:

12.353m

(2) 25. Let θ be the angle in standard position with terminal side containing the point $(1, -2)$. Find the exact value of $\sin\theta$ and $\sec\theta$.

Solution:

$$\sin\theta = \frac{-2}{\sqrt{5}}, \ \sec\theta = \sqrt{5}$$

(2) 26. For the angle $\theta = 1000°$, find:

(a) the reference angle,

Solution:

$80°$

(b) the value of $\cot\theta$ accurate to 4 decimal places.

Solution:

-0.1763

(2) 27. Find the exact value of two angles θ in the interval $[0, 2\pi)$ with $\sin\theta = -\frac{1}{2}$.

Solution:

$$\frac{7\pi}{6}, \frac{11\pi}{6}$$

(1) 28. Find the exact value of two angles θ in the interval $[0°, 360°)$ with $\tan\theta = 0$.

Solution:

$$0°, 180°$$

(4) 29. Prove the identities:

(a) $\sec^2 x \cot x = \tan x + \cot x$

Solution: There are many ways to prove it, here shows only one method.

$$\frac{1}{\cos^2 x} \cdot \frac{\cos x}{\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$
$$\frac{1}{\cos x \sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$
$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x} \quad (\text{recall } \sin^2 x + \cos^2 x = 1)$$

(b) $1 - \frac{\sin^2 x}{1+\cos x} = \cos x$

Solution: There are many ways to prove it, here shows only one method.

$$\frac{1+\cos x}{1+\cos x} - \frac{\sin^2 x}{1+\cos x} = \cos x$$
$$\frac{1+\cos x - \sin^2 x}{1+\cos x} = \cos x \quad (\text{recall } 1 - \sin^2 x = \cos^x)$$
$$\frac{\cos^2 x + \cos x}{1+\cos x} = \cos x$$
$$\frac{\cos x(\cos x + 1)}{1+\cos x} = \cos x$$
$$\cos x = \cos x$$

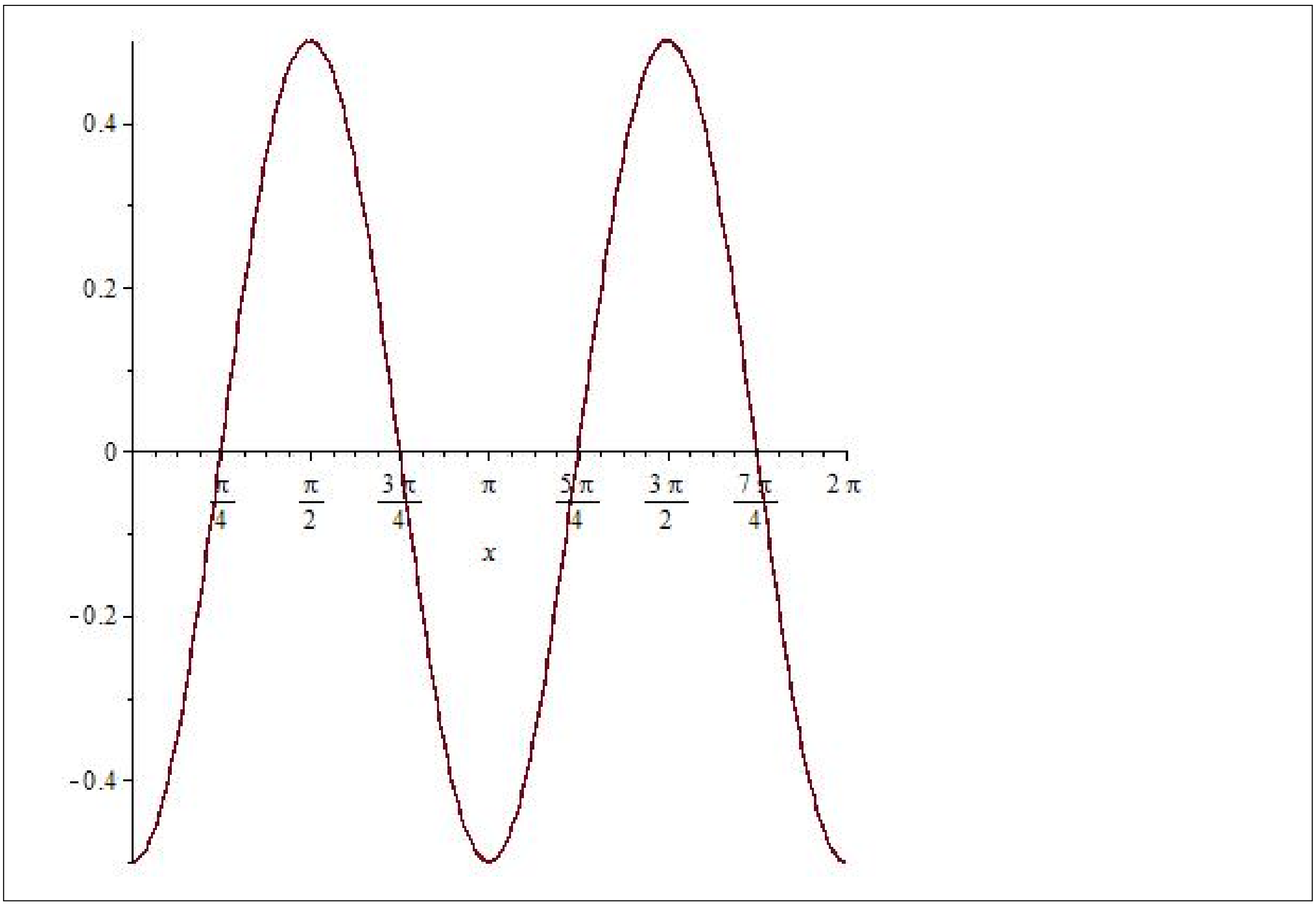
(4) 30. Let $y = -\frac{1}{2}\cos(2x)$.

(a) Find the amplitude and period.

Solution: Amplitude: $\frac{1}{2}$; period: π.

(b) Sketch two cycles of this function.

Solution:



(3) 31. For the triangle $\triangle ABC$ with sides $a = 5$ and $b = 7$ and angle $A = 40°$, find the angle B and side c (round your answers to two decimal places).

Solution:

$$B = 64.15°,\ c = 7.54$$

(2) 32. For the triangle $\triangle ABC$ with sides $a = 3, b = 7$ and $c = 7$, find the angles A and B (round your answers to two decimal places).

Solution:

$$A = 24.75°,\ B = 77.63°$$