(Marks)

(3) 1. Evaluate each expression without a calculator. (Show your work):

(a) $\frac{75!4!}{74!5!}$ (b) $8^{C}5$ (c) $9^{P}2$

(4) 2. Seven students have three tickets to a concert. Assuming that no student can get more than one ticket, how many ways are there for them to distribute the tickets if…

(a) …the tickets are for assigned seats?

(b) …the tickets are for general admission? (i.e., No assigned seats.)

(6) 3. Let $U = \{!, @, \#, \$, \%, \wedge, \&, *, (,)\}$ be a universal set.
Let $A = \{!, @, \#, \$, \%\}$, $B = \{\wedge, \&, *, (,)\}$, and $C = \{!, \#, \%, \&, (\}$.

(a) Find $B \cap \overline{C}$

(b) Find $(B \cap C) \cup (\overline{C} \cap C)$

(c) Find $\overline{A \cup B}$

(d) Find $A \cap (B \cup C)$

(e) How many proper subsets does A have?

(f) Are A and B equivalent? Explain.

(4) 4. Use a Venn diagrams to illustrate the following identity: $(A \cap B) \cup (B \cap \overline{C}) = B \cap (A \cup \overline{C})$. Make sure you indicate which hatched areas belong to which sets.

(4) 5. Name each set property used in the following simplification:

$\overline{A \cap B} \cap B = (\overline{A} \cup \overline{B}) \cap B = (\overline{A} \cap B) \cup (\overline{B} \cap B) = (\overline{A} \cap B) \cup \emptyset = (\overline{A} \cap B)$

(5) 6. Use truth tables to determine if the following statements are equivalent:

$$p \overline{\wedge} (q \overline{\vee} {\sim} r) \text{ and } ({\sim} p \overline{\vee} q) \overline{\wedge} r$$

(4) 7. Use truth tables to determine if the following is a tautology, a contradiction, or neither:

$$(p \leftrightarrow q) \rightarrow (p \underline{\vee} q)$$

(5) 8. Given the statement: $S :=$"If there's a fire, then the alarm goes off."

(a) State the converse of S.

(b) State the inverse of S.

(c) State the contrapositive of S.

(d) Of the four statements above, which are contradicted by a false alarm? (i.e. which are false when there is an alarm but no fire?)

(5) 9. Use truth tables to determine if the following argument is valid:

H: If I lose my keys, either I break a window or I sleep outside.
If I break a window, then I sleep outside.

C: If I lose my keys, then I sleep outside.

(3) 10. Use Venn Diagrams to determine if the following argument is valid. Be sure to name and label your sets.

H: All lovers are sinners.
No sinners are grinners.

C: No lovers are grinners.

(3) 11. Create a Boolean Table for the following expression: $\overline{(A + B)}(A + C)$

(2) 12. Draw a network diagram that represents the following Boolean Expression: $D + C[\overline{D} + (A + B)]$

(6) 13. Use network properties to simplify the expression. Use one property in each step and indicate the name of each property that you use.

(a) $(AB)C + BC$

(b) $A + \overline{A + B}$

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(2) 14. State both identity properties for networks.

(6) 15. Given the system $\begin{array}{rcl} 3x - y & = & 6 \\ -6x + 2y & = & 18 \end{array}$

(a) Graph each line.

(b) Solve the system algebraically by substitution or elimination.
(Is this solution consistent with the result from part (a)?)

(c) Is the system consistent? Explain.

(10) 16. Given: $A = \begin{bmatrix} 2 & -5 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \\ -3 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 3 & -2 \\ -3 & 1 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 5 \\ 2 & -3 \end{bmatrix}$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find each of the following, if possible. If an operation is not possible, say why.

(a) $3I_3 + AA^{-1}$

(b) DB

(c) D^2

(d) $I_3 - C$

(e) CI_3

(f) $C^T - 2B$

(g) BC

(6) 17. Given $A = \begin{bmatrix} 2 & -3 & 1 \\ -1 & 4 & -1 \\ 4 & 0 & 1 \end{bmatrix}$ find A^{-1} using elementary row operations and verify your answer.

(3) 18. Given $A = \begin{bmatrix} 8 & -4 \\ -2 & 1 \end{bmatrix}$ show that A^{-1} does not exist.

(5) 19. Solve, if possible, the following system using matrices and row reduction.

$$\begin{array}{rcrcrcr} -x & + & 2y & + & z & = & -6 \\ & & y & + & 3z & = & 3 \\ 2x & - & 2y & + & 4z & = & 5 \end{array}$$

(5) 20. Solve, if possible, the following system using matrices and row reduction.

$$\begin{array}{rcrcrcr} x & + & 3y & - & 2z & = & -2 \\ 3x & + & 8y & - & 4z & = & -6 \\ -x & + & 4y & - & 7z & = & 7 \end{array}$$

(4) 21. Consider the system of linear equations

$$\begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & 4 \\ 3x & - & 5y & + & 14z & = & -2 \\ 2x & - & 4y & + & 7z & = & 5 \end{array}$$

(a) Write the system in matrix form $AX = B$.

(b) Show that $A^{-1} = \begin{bmatrix} 21 & 2 & -13 \\ 7 & 1 & -5 \\ -2 & 0 & 1 \end{bmatrix}$ is the inverse matrix of A.

(c) Use A^{-1} to solve given system of linear equations.

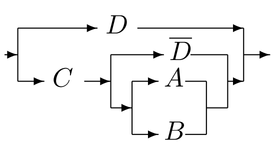
(2) 22. Give all solutions of the system $AX = B$, whose augmented matrix can be reduced to $\left[\begin{array}{rrr|r} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

(Marks)

(3) 23. Use mathematical induction to prove that the following statement is true for all positive integers n:

$$6 + 10 + 14 + \cdots + (4n + 2) = 2n(n + 2)$$

ANSWERS:

1. (a) 15 (b) 56 (c) 72
2. (a) $7^{P}3 = 210$ (b) $7^{C}3 = 35$
3. (a) $\{\wedge, *,)\}$, (b) $\{\&, (\}$ (c) $\{\}$ (d) $\{!, \#, \%\}$ (e)$2^5 - 1 = 31$
   (f) Yes, they have the same number of elements.
4.
5. deMorgan, distributive, complement, identity
6. Neither
7. Equivalent
8. (a) "*If the alarm goes off, then there is a fire.*"
   (b) "*If there is no fire, then the alarm doesn't go off.*"
   (c) "*If the alarm doesn't go off, then there is no fire.*"
   (d) Converse and inverse
9. Valid argument
10. Valid argument
11. All zeros except the case when A and B are both 0, and $C = 1$.
12. 

13. (a) BC (b)$A + \overline{B}$
14. $A + 0 = A$ and $A \cdot 1 = A$
15. (a) Parallel lines (b) No solution (c) System with no solution, therefore inconsistent .
16. (a) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (b) undefined (c) $\begin{bmatrix} 11 & -10 \\ -4 & 19 \end{bmatrix}$ (d) Undefined
    (e)$\begin{bmatrix} 2 & 3 & -2 \\ -3 & 1 & 4 \end{bmatrix}$ (f) $\begin{bmatrix} -2 & -5 \\ 3 & 5 \\ 4 & -6 \end{bmatrix}$ (g) $\begin{bmatrix} 1 & 7 & 0 \\ 6 & -2 & -8 \\ -21 & -4 & 26 \end{bmatrix}$
17. $A^{-1} = \begin{bmatrix} 4 & 3 & -1 \\ -3 & -2 & 1 \\ -16 & -12 & 5 \end{bmatrix}$ and $AA^{-1} = I$
18. No inverse, A can not be reduced to I.
19. No solution
20. $\begin{aligned} x &= -6 \\ y &= 2 \\ z &= 1 \end{aligned}$
21. a) $\begin{bmatrix} 1 & -2 & 3 \\ 3 & -5 & 14 \\ 2 & -4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$ b) $AA^{-1} = I$ (c) $X = A^{-1}B = \begin{bmatrix} 15 \\ 1 \\ -3 \end{bmatrix}$

(Marks)

22. $x = 1 + 2t$
$y = -2 - t$
$z = t\,(any\ number)$

23. $P_1 : 6 = 2(1)(1+2)$ true
Assuming that P_k is true, P_{k+1} : LHS $= 2k(k+2) + (4k+6) = 2k^2 + 8k + 6 = 2(k+1)(k+3) =$ RHS.