

1. Evaluate each of the following integrals, without the use of integration tables

$$(3) \text{ a) } \int \frac{5x^2 - x^3 \sec^2 x + x^3 e^{3x} + x^3 \cos x}{x^3} dx$$

$$(3) \text{ b) } \int_0^1 5x\sqrt{x^2+3} dx \quad (3) \text{ c) } \int \frac{x}{x+2} dx \quad (3) \text{ d) } \int x^2 \ln x dx \quad (3) \text{ e) } \int \frac{5x^2 - 8x + 2}{x(x-1)^2} dx$$

$$(3) \text{ f) } \int \frac{20x^2 - 27x + 11}{4x-3} dx \quad (3) \text{ g) } \int xe^{2x} dx \quad (3) \text{ h) } \int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

2) (2 marks) Find the demand function given $\frac{dR}{dx} = x^2 - 2x + 3$

3) (4 marks) Given the functions $f(x) = 4x - x^2$ and $g(x) = x^2 - 6x + 8$

a) Sketch the region R bounded by the graphs $f(x)$ and $g(x)$

b) Find the area of the region R

4) (5 marks) Given the demand function $p_1 = 200 - x^2$ and the supply function $p_2 = 6x + 160$

a) Find the equilibrium point.

b) Sketch and identify the regions representing the consumer and producer surpluses.

c) Evaluate the producer's surplus.

5) (4 marks) Use Simpson's Rule to approximate $\int_2^6 \frac{10}{\sqrt{x^2+4}} dx$ Use $n=4$ and

round your answer to four decimal places.

6) (6 marks) Use the table of integrals to solve the following.

In each case, state the formula number and justify its use

$$\text{a) } \int \frac{3x}{(2+x^2)(5+x^2)} dx \quad \text{b) } \int \frac{\sqrt{x^2+4x+5}}{x+2} dx$$

7) (2 marks) Determine whether $y = x^3 - \frac{5}{2}x$ is a solution to the differential equation $xy' - 3y = 5x$

8) (8 marks) Solve the following differential equations.

a) $y' = 2xy + 3x^2y$ given $y(2) = 1$ and $y > 0$

b) $y' = \frac{3x^2}{\sqrt{y}}$ given $y(1) = 9$

9)(4 marks) The rate of increase of the population P of a city is proportional to the product of the population size P and the square of time t in Years. Initially, the population is 100,000 and in 3 years it is 300,000. How long will it take for the population to be 1,200,000?

10)(8 marks) Evaluate the following limits :

$$a) \lim_{x \rightarrow 0} \frac{x^2 - e^{2x} + \cos x}{3x^2 + \sin 3x} \quad b) \lim_{x \rightarrow \infty} \frac{(3x-4)^3}{9x^2 - 5x^3}$$

11)(8 marks) Determine whether the following improper integrals converge or diverge.

If the integral converges, find its value.

$$a) \int_5^{\infty} \frac{x}{\sqrt{x^2 - 16}} dx \quad b) \int_0^2 \frac{1}{x^3} dx$$

12)(3 marks) Consider the sequence $\left\{ \frac{3}{4}, \frac{5}{7}, \frac{7}{10}, \frac{9}{13}, \dots \right\}$

a) Give the 7th term of the sequence.

b) Find an expression for the n^{th} term of the sequence.

13)(8 marks) Determine if the following sequences converge or diverge. If the sequence converges, find its limit.

$$a) a_n = \frac{3^n - 1}{3^n} \quad b) a_n = \frac{2(n+2)!}{(n+1)!}$$

14)(8 marks) Determine if the following series converge or diverge. If the series converges, find the sum.

$$a) \sum_{n=1}^{\infty} 2(0.99)^n \quad b) \sum_{n=1}^{\infty} \frac{1}{12} \left(\frac{9}{4} \right)^n$$

15)(3 marks) A deposit of 35 dollars is made at the beginning of each month for 4 years into an account that pays 1.5% interest, compounded 12 times a year. Find the balance in the account at the end of the 4 years.

16)(3 marks) Given the number $5.\overline{011}$, express it using a geometric series. Find the sum of the geometric series to write the number as the ratio of two integers.

1 a) $5\ln|x| - \tan x + \frac{e^{3x}}{3} + \sin x + C$

b) = 4.673

c) $x + 2 - 2\ln|x + 2| + C$

d) $\frac{x^3}{3} \ln x - \frac{x^3}{9} + C$

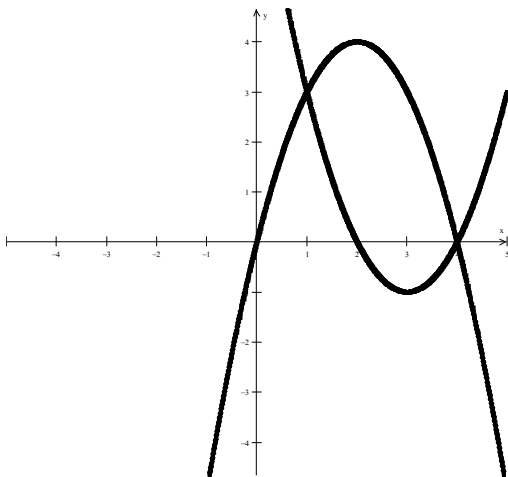
e) $2\ln|x| + 3\ln|x - 1| + \frac{1}{x - 1} + C$

f) $\frac{5x^2}{2} - 3x + \frac{1}{2}\ln|4x - 3| + C$

g) $\frac{x}{2}e^{2x} - \frac{1}{4}\int e^u du \dots \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$ h) $2e^2 - 2e^1 = 9.34$

2) $p = \frac{x^2}{3} - x + 3$

3) 9 units



4) \$48.

5) 9.3702

$$6) \frac{1}{2} \ln \left| \frac{2+x^2}{5+x^2} \right| + C$$

b) Formula #24

$$\sqrt{(x+2)^2+1} - \ln \left| \frac{1+\sqrt{(x+2)^2+1}}{x+2} \right| + C$$

7) it is a solution

$$8) y = e^{x^2+x^3-12}$$

$$b) y = \sqrt[3]{\left(\frac{3x^3}{2} + \frac{51}{2}\right)^2}$$

9) 3.9 years

$$10) a) \frac{-2}{3}$$

$$b) \frac{-27}{5}$$

11) **ans:** ∞ divergesb) $\infty \dots$ diverges

$$12) a) a_7 = \frac{15}{22}$$

$$b) a_n = \frac{1+2n}{1+3n}$$

13) a) 1 convergent

$b) \infty$ **divergent**

14)a) **198**

$b)$. **divergent**

15) \$1732.47

16) $\frac{5006}{999}$