JOHN ABBOTT
CEGEP/COLLEGE

Department of Mathematics

Final Examination

December 11, 2014
14:00–17:00

Mathematical Models
201-115-AB

Instructors: R.Masters and C.Morris

Student name: _____

Student number: _____

Instructor: _____

Instructions

1. Do not open this booklet before the examination begins.
2. Check that this booklet contains 5 pages, excluding this cover page and the formula sheet.
3. Write all of your solutions in this booklet and show all supporting work.
4. If the space provided is not sufficient, continue the solution on the opposite page.

(4) **1.** Consider $f(x) = 5\cos\left(\frac{\pi}{2}x + \pi\right) + 1$

(a) What is the amplitude of the function?

(b) What is its period?

(c) What is its frequency?

(d) What is its phase shift?

(3) **2.** A soccer ball has a circumference of approximately 53.5 cm. Find its volume.

3. Perform the indicated operations. Express the result in polar form.

(3) (a) $\dfrac{4\left[\cos 75° + j\sin 75°\right]}{24\left[\cos 35° + j\sin 35°\right]}$

(3) (b) $5\left[\cos 95° + j\sin 95°\right] - 8\left[\cos 210° + j\sin 210°\right]$

4. Evaluate and express your answer in the form $a + bj$.

(3) (a) $(5j^7 - 3j^{10} + 2j)(3j^{15} - 4j^{24})$

(3) (b) $\dfrac{-4}{2j+5} + \dfrac{3+j}{j^2}$

(5) **5.** Solve the following trigonometric equation for all values of x such that $0 \le x < 2\pi$.

$$3\cos^2\left(\frac{x}{2}\right) + 2\cos^2(x) = 2 - \sin^2\left(\frac{x}{2}\right)$$

(5) **6.** An airplane traveling along a vector heading 50° North of West must reach a destination located 3200 km away from it's current position in exactly 5 hours. The plane flies against a head wind of 70 km/h in a direction of 80° South of East. What velocity should the airplane's motor achieve in order meet this objective?

(4) **7.** Use DeMoivre's Theorem to find the cube roots of $-j$. Express your answers in rectangular form $x + yj$.

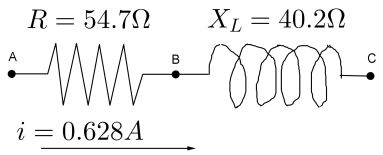
8. Solve the following equations for x.

(3) (a) $\log_{\sqrt{15}}(x-3) + \log_{\sqrt{15}}(x-1) = 2$

(3) (b) $7^{2-3x} = \left(\dfrac{1}{49}\right)^{2x+3}$

(3) **9.** Solve the following system for y using Cramer's rule.(No marks will be awarded if Cramer's rule is not used)

$$\begin{array}{rcrcrcr} 4x & + & 9y & & & = & -1 \\ & & 3y & - & z & = & -5 \\ 8x & & & + & 3z & = & 10 \end{array}$$



10. Use the diagram above to determine :

(2) (a) The (complex) voltage across the resistor (between points A and B).

(2) (b) The (complex) voltage across the inductor (between points B and C).

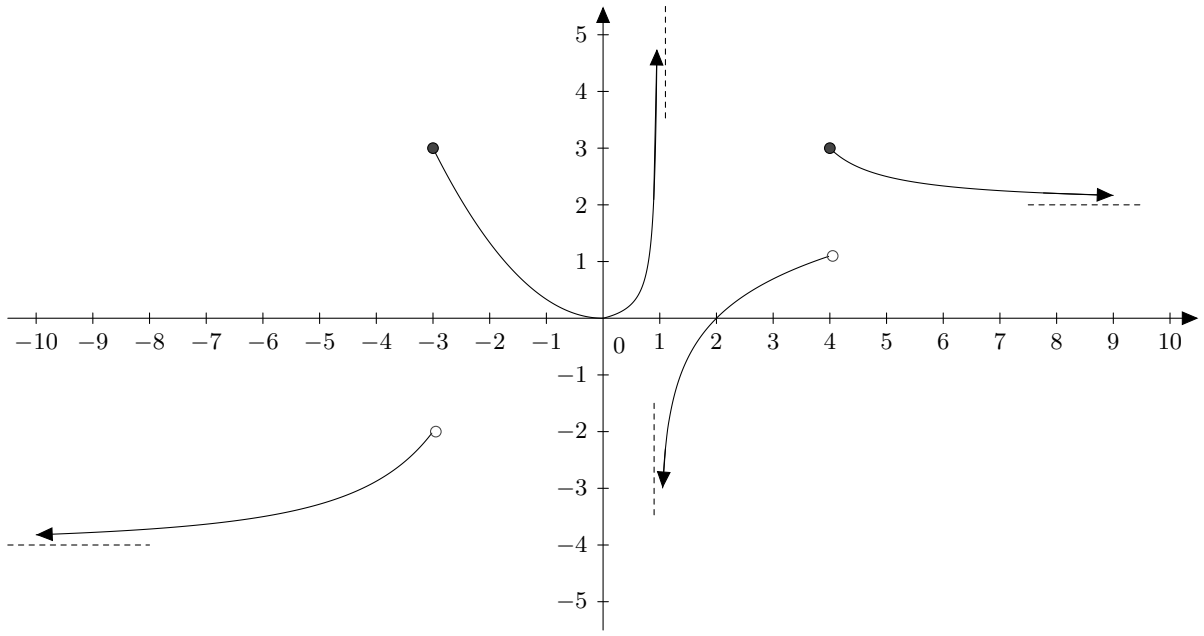
(2) (c) The magnitude of the voltage across the combination (between points A and C).

(2) (d) By what angle (in degrees) does the voltage lead the current.

(5) **11.** Solve the following linear system.

$$\begin{array}{rcrcrcr} 3x & - & 2y & - & z & = & 8 \\ & & 6y & + & 5z & = & 13 \\ x & + & 5y & + & 2z & = & 3 \end{array}$$

(5) **12.** For the function $f(x)$ given in the diagram below, find each of the following limits. If the limit does not exist, write DNE or $-\infty$ or ∞ where appropriate. If the function is undefined at a point write UND.



(a) $\lim_{x\to-\infty} f(x) =$ _____

(b) $\lim_{x\to\infty} f(x) =$ _____

(c) $\lim_{x\to-3} f(x) =$ _____

(d) $\lim_{x\to1^+} f(x) =$ _____

(e) $\lim_{x\to4^-} f(x) =$ _____

(f) $\lim_{x\to4^+} f(x) =$ _____

(g) $f(-3) =$ _____

(h) $f(4) =$ _____

(i) List points of discontinuity

(4) **13.** Find the equation of the Tangent line to the curve $f(x) = x^2 - 5x + 4$ at the point $(2, -2)$.

14. Given the function $f(x) = \cos^2 x$ Find,

(2) (a) $f'(x) =$

(2) (b) $f''(x) =$

(2) (c) $f''(\pi/2) =$

15. Evaluate the following limits if possible.

(3) 1. $\lim\limits_{x\to 2} \dfrac{x^2 + 2x - 8}{x^2 - 2x}$

(3) 2. $\lim\limits_{x\to \infty} \dfrac{3x^3 + 5x - 2}{-10x^4 + 12x^2}$

(3) 3. $\lim\limits_{x\to 3^-} \dfrac{x + 3}{|x - 3|}$

(3) 4. $\lim\limits_{x\to -\infty} \dfrac{\sqrt{x^2 - 16}}{4x - 12}$

(3) **16.** Given $f(x) = e^x - 2x$, find the x-coordinate(s) where the curve has horizontal tangents.

17. Find y' by differentiating the following;(**Do not simplify your answer**)

(3) 1. $y = \sqrt[7]{x} - 3x^5 + 2^x - e^{\sin(e)}$

(3) 2. $y = \log_6(\sin x) + \dfrac{5x}{x + 4}$

(3) 3. $y = \tan(4x) + e^{x^2} - \ln(9x + 1)$

(3) 4. $\cos(x - y) = x^2 y - 15$

(3) 5. $y = (x + 2)(x - 1)^x$ Hint: use logarithmic differentiation

Mathematical Models 115: Formula Sheet Fall 2014

1. Volume of sphere: $V = \frac{4}{3}\pi r^3$
2. Volume of cylinder: $V = \pi r^2 h$
3. Volume of cone: $V = \frac{1}{3}\pi r^2 h$
4. Rectangular: $x + yj$
5. Polar: $r(\cos\theta + j\sin\theta) = r\angle\theta$
6. Exponential: $re^{j\theta}$
7. $x = r\cos\theta \quad y = r\sin\theta$
   $r = \sqrt{x^2 + y^2}$
   $\tan\theta = \frac{y}{x}$
8. DeMoivre's Theorem:
   $[r(\cos\theta + j\sin\theta)]^n = r^n(\cos n\theta + j\sin n\theta)$
9. $V_R = IR$
   $V_C = IX_C$
   $V_L = IX_L$
   $V_{RLC} = IZ$
   $Z = R + j(X_L - X_C)$
   $Z = \sqrt{R^2 + (X_L - X_C)^2}$
   $\theta = \tan^{-1}\frac{X_L - X_C}{R}$
10. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
    $\log_b(xy) = \log_b x + \log_b y$
    $\log_b x^p = p\log_b x$
    $\log_b b = 1$
    $\log_b(1) = 0$
    $x = \log_b y \Leftrightarrow b^x = y$
11. $c^2 = a^2 + b^2 - 2ab\cos C$
    $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
12. $(x^3 \pm y^3) = (x \pm y)(x^2 \mp xy + y^2)$
13. $\sin^2 x + \cos^2 x = 1$
    $1 + \tan^2 x = \sec^2 x$
    $1 + \cot^2 x = \csc^2 x$
14. $\sin 2x = 2\sin x\cos x$
    $\cos 2x = \cos^2 x - \sin^x$
15. $\sin^2 x = \frac{1 - \cos 2x}{2}$
    $\cos^2 x = \frac{1 + \cos 2x}{2}$
16. $\frac{d}{dx}(fg) = f'g + fg'$
17. $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$
18. $\frac{d}{dx}(x^n) = nx^{n-1}$
19. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$
20. $\frac{d}{dx}(cf) = cf'$

Answers Fall 2014

1. (a) $a = 5$
   (b) $p = 4$
   (c) $f = \frac{\pi}{2}$
   (d) $x = -2$
2. $2581.5\ cm^3$
3. (a) $\frac{1}{6}[\cos 40° + j \sin 40°]$
   (b) $11.02[\cos 53.92° + j \sin 53.92°]$
4. (a) $-21 + 3j$
   (b) $-\dfrac{107}{29} - \dfrac{21}{29}j$
5. $x = \dfrac{\pi}{2}; \dfrac{2\pi}{3}; \dfrac{4\pi}{3}; \dfrac{3\pi}{2}$
6. 701.50 km/h at 52.86° NW
7. $j;\ -\frac{\sqrt{3}}{2} - \frac{1}{2}j;\ \frac{\sqrt{3}}{2} - \frac{1}{2}j$
8. (a) $x = 6$
   (b) $x = -8$
9. $y = \frac{1}{3}$
10. (a) $34.4v$
    (b) $25.2v$
    (c) $42.6v$
    (d) $36.3°$
11. $x = 3;\ y = -2;\ z = 5$
12. (a) -4
    (b) 2
    (c) DNE
    (d) $-\infty$
    (e) 1
    (f) 3
    (g) 3
    (h) 3
13. $y = -x$
14. (a) $-2 \sin x \cdot \cos x$
    (b) $-2 \cos x$
    (c) 2
15. (a) 3
    (b) 0
    (c) ∞
    (d) $-\frac{1}{4}$
16. $x = \ln 2$
17. (a) $\dfrac{1}{7x^{6/7}} - 15x^4 + 2^x \ln 2$
    (b) $\dfrac{\cot x}{\ln 6} + \dfrac{20}{(x+4)^2}$
    (c) $4 \sec^2(4x) + 2xe^{x^2} - \dfrac{9}{9x+1}$
    (d) $\dfrac{2xy + \sin(x-y)}{-x^2 + \sin(x-y)}$
    (e) $(x+2)(x-1)^x \left(\dfrac{1}{x+2} + \ln(x-1) + \dfrac{x}{x-1} \right)$