December 18th 2014 Final Exam
ALGEBRA and TRIGONOMETRY
MATH 201-015-50

1. Simplify the following expressions completely. Eliminate any negative exponents and simplify the radicals.

   (a) $\dfrac{y^{-2}x^{-2}}{x^{-7}y}\left(\dfrac{1}{x}\right)^{2}\left(\dfrac{x}{y^{2}}\right)^{-2}$

   (b) $\dfrac{\sqrt[3]{64a^{6}b^{7}}}{\sqrt{16a^{4}b^{5}}}$

2. Rationalize the denominator and simplify $\dfrac{2x}{\sqrt{2x}+\sqrt{x}}$

3. Factor each of the following completely

   (a) $6x^{2}-4x-2$

   (b) $27y-y^{4}$

   (c) $13x^{2}-13+yx^{2}-y$

4. Perform the following operations and simplify the expressions.

   (a) $\dfrac{x^{2}y^{2}-xy^{2}}{xy+y}\div\dfrac{x^{2}-1}{x^{2}-2x+1}$

   (b) $\dfrac{3x-7}{3x^{2}-15x+18}-\dfrac{3x-1}{3(x^{2}-2x-3)}$

   (c) $\dfrac{\frac{1}{x-1}-\frac{1}{x+y}}{\frac{1}{x+y}+\frac{1}{x-1}}$

5. Perform the long division, clearly stating the quotient and the remainder: $\dfrac{2x^{3}-4x-2}{x+3}$

6. Consider the points $A(-2,5)$ and $B(1,-1)$.

   (a) Find the distance between A and B.

   (b) Find the coordinates of the point halfway between A and B

   (c) Find an equation of the line passing through the points A and B.

(d) Find the equation of the line passing through B and perpendicular to the line given by $y = \frac{2}{3}x - 4$.

(e) Find an equation of the line parallel to the $y-$axis and passing through A.

7. Solve for x in each of the following:

(a) $3 - 2(x - 5) = 3x - (x + 7)$

(b) $4x^2 + 3x = 48 - 13x$

(c) $x^2 - 8x - 14 = 0$

(d) $x^4 + 10x^3 = 2x^3 + x^2 + 8x$

(e) $\sqrt{x + 5} - \sqrt{5 - x} = 2$

(f) $x - (2 - 4x) \geq \dfrac{x - 3}{2} + 1$

(g) $\dfrac{1}{x + 4} + 1 = \dfrac{2}{x + 1} - \dfrac{6}{x^2 + 5x + 4}$

8. Find the domain of $\frac{\sqrt{x+1}}{5-x}$

9. State the domain of the function $y = \frac{x+2}{x-3}$, and sketch the graph showing any intercepts and asymptotes.

10. Find the intercepts, the axis of symmetry, the vertex and the range of $y = -x^2 + 2x + 8$. Sketch the graph of the function, showing these features.

11. Let $f(x) = \frac{2x}{x-3}$ and $g(x) = 7 - 5x$

(a) Find and simplify $f(g(x))$

(b) Find $f^{-1}(x)$

12. Let $f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq 3 \\ x - 6 & \text{if } x > 3 \end{cases}$

(a) Sketch the graph of $f(x)$.

(b) What is the range of $f(x)$?

13. Use a calculator to evaluate $\log_5 27$ to 4 decimal places.

14. Find exact values for x in the following equations:

(a) $\dfrac{4^{x-3}-6}{2}=5$

(b) $5+3\log_2(4-3x)=2$

15. Solve $3^{x+1}=5^x$. Give at least 4 decimal places.

16. Express $\ln\left(\frac{(x-3)^2\sqrt{x^2+1}}{4(x+2)^5}\right)$ in terms of the simplest possible logarithms.

17. Express as a single logarithm: $\frac{2}{3}\ln x-5\ln z-3\ln y$.

18. Find the domain and range of $y=\log_2(x+4)$. Sketch the graph, indicate any intercepts and asymptotes.

19. Sketch the graph of $y=1-3^x$. Indicate the intercepts and asymptotes.

20. Find the value after seven years of \$6000 invested at 5.5% interest compounded monthly.

21. Let θ be the angle in standard position whose terminal side contains the point $(-12,-5)$. Find

    (a) the exact value of $\cot\theta$

    (b) the exact value of $\csc\theta$.

22. An angle θ in standard position has terminal side in the third quadrant and satisfies $\cos\theta=-\dfrac{2}{3}$. Find the **exact value** of $\tan\theta$.

23. Find the exact values of the reference angle for $\frac{7\pi}{13}$ both in radians and degrees.

24. Find the exact value of two angles θ in degrees in the interval $[0°, 360°)$, where $\sec\theta=-\sqrt{2}$

25. Find the exact value of the angle θ in radians in the interval $[0, 2\pi)$, where $\tan\theta=\sqrt{3}$ and θ is in quadrant III.

26. Let $y=-3\sin\left(\dfrac{x}{4}\right)$.

    (a) Find its amplitude and period.

    (b) Graph two cycles of this function.

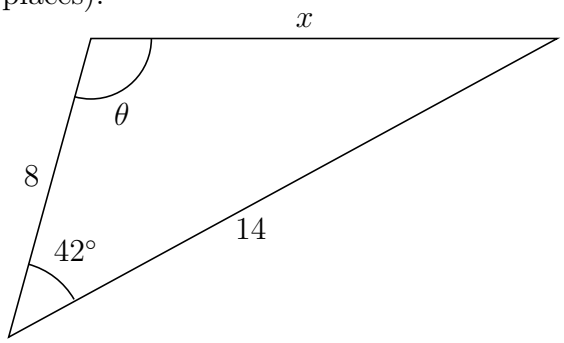
27. Prove the following identities:

(a) $\sec\theta - (\sin\theta)(\tan\theta) = \cos\theta$

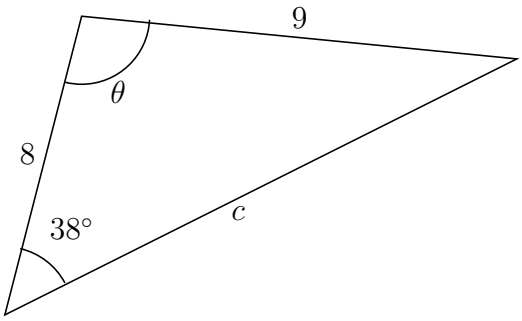
(b) $\dfrac{1}{1-\cos\theta} - \dfrac{1}{1+\cos\theta} = 2(\csc\theta)(\cot\theta)$

28. Hana is watching a hot-air balloon rise while standing 1 kilometer away from the point on the ground directly below the balloon. She makes two measurements of the angle of elevation from herself to the balloon. The first is 20 degrees. After watching the balloon rise further, she makes a second measurement, which is 50 degrees. How far did the balloon rise during the interval of time between her first and second measurements ?

29. For the triangle shown below, find the length x first and then find the angle θ in degrees. (two decimal places).



30. Find the angle θ and the length of side c in the triangle below. (two decimal places).



Answers

1. (a) xy; (b) $\frac{1}{\sqrt[6]{b}}$;
2. $2\sqrt{2x} - 2\sqrt{x}$;
3. (a) $2(3x+1)(x-1)$; (b) $y(3-y)(9+3y+y^2)$; (c) $(13+y)(x-1)(x+1)$;
4.(a) $\frac{xy(x-1)^2}{(x+1)^2}$; (b) $\frac{1}{(x-2)(x+1)}$; (c) $(y+1)(2x+y-1)$;
5. quotient is $2x^2 - 6x + 14$, remainder is -44;
6. (a) $3\sqrt{5}$; (b) $(-\frac{1}{2}, 2)$; (c) $y = -2x + 1$; (d) $y = -\frac{3}{2}x + \frac{1}{2}$; (e) $x = -2$;
7. (a) $x = 5$; (b) $x = 2, x = -6$; (c) $x = 4+\sqrt{30}, x = 4-\sqrt{30}$; (d) $x = 0, x = 1, x = -1, x = -8$;
(e) $x = 4$, ($x = -4$ is inadmissable); (f) $x \geq \frac{1}{3}$; (g) $x = -3$, ($x = -1$ is inadmissable);
8. $[-1, 5) \cup (5, \infty)$;
9. domain is all x such that $x \neq 3$, x-intercept is at $(-2, 0)$, y-intercept is $(0, -\frac{2}{3})$ x-horizontal asymptote is $y = 1$, vertical asymptote is $x = 3$;
10. x-intercepts are $(4, 0)$ and $(-2, 0)$, y-intercept $(0, 8)$, vertex $(1, 9)$, range $(-\infty, 9]$, axis of symmetry $x = 1$;
11. (a) $\frac{14-10x}{4-5x}$; (b) $\frac{3x}{x-2}$;
12. (a) graph is below (b) $[-4, \infty)$;
13. 2.0478;
14. (a) $x = 5$; (b) $x = \frac{7}{6}$;
15. $x = \frac{\ln 3}{\ln 5 - \ln 3} = 2.15066$;
16. $2\ln(x-3) + \frac{1}{2}\ln(x^2+1) - \ln 4 - 5\ln(x+2)$;
17. $\ln\left(x^{2/3}z^{-5}y^{-3}\right)$;
18. domain is $(-4, \infty)$, range is $(-\infty, \infty)$, vertical asymptote is $x = -4$;
19. x- and y-intercept $(0, 0)$, horizontal asymptote is $y = 3$;
20. $8,809.93;
21. (a) $\cot\theta = \frac{12}{5}$; (b) $\csc\theta = -\frac{13}{5}$;
22. $\tan\theta = \frac{\sqrt{5}}{2}$;
23. reference angle in radians is $\frac{6\pi}{13}$, in degrees is $\left(\frac{1089}{13}\right)^\circ$;
24. $\theta = 135^\circ$, $\theta = 225^\circ$;
25. $\theta = \frac{4\pi}{3}$;
26. (a) amplitude is 3, period is 8π; (b) graph is on last page
27. (a), (b) proofs on next page
28. 827.78 meters;
29. $x = 9.67$, $\theta = 104.39^\circ$
(the supplement 75.61°, is incorrect !);
30. $\theta = 108.82^\circ$, $c = 13.84$.

Proof for 27. (a)

$$\begin{aligned}
\sec\theta - (\sin\theta)(\tan\theta) &= \frac{1}{\cos\theta} - \left(\frac{\sin\theta}{1}\right)\left(\frac{\sin\theta}{\cos\theta}\right) \\
&= \frac{1-\sin^2\theta}{\cos^2\theta} \\
&= \frac{\cos^2\theta}{\cos\theta} \\
&= \cos\theta
\end{aligned}$$

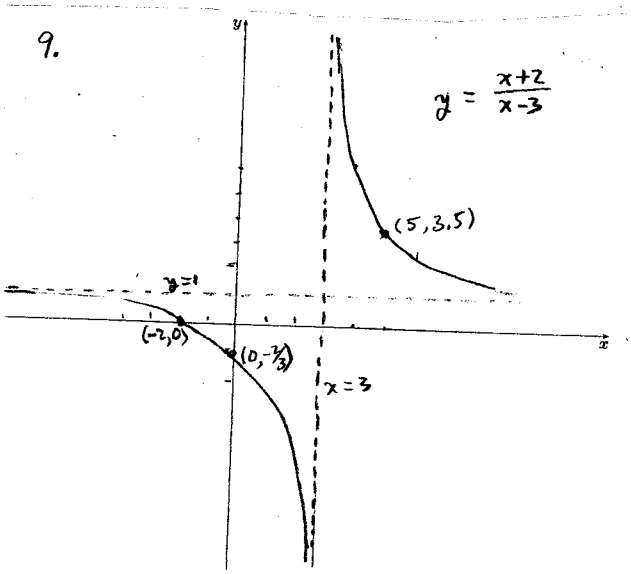
Proof for 27. (b)

$$\begin{aligned}
\frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta} &= \frac{(1+\cos\theta)-(1-\cos\theta)}{1-\cos^2\theta} \\
&= \frac{2\cos\theta}{\sin^2\theta} \\
&= 2\left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) \\
&= 2(\csc\theta)(\cot\theta)
\end{aligned}$$

Graphs

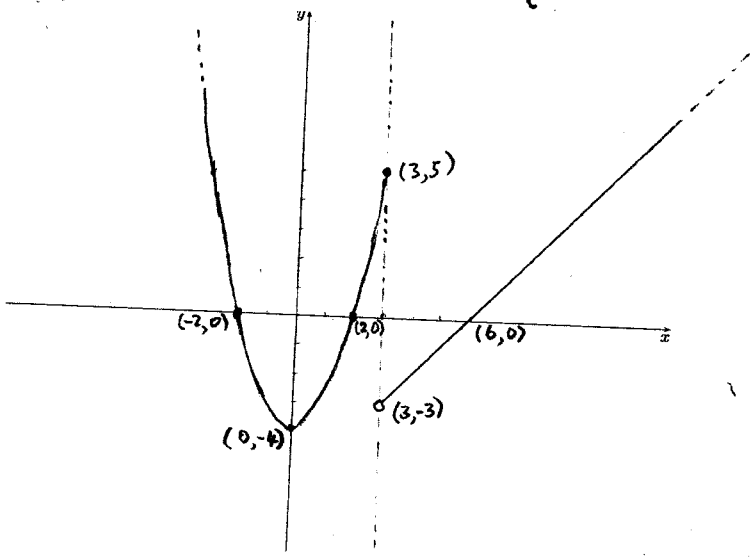
9.

$y = \frac{x+2}{x-3}$



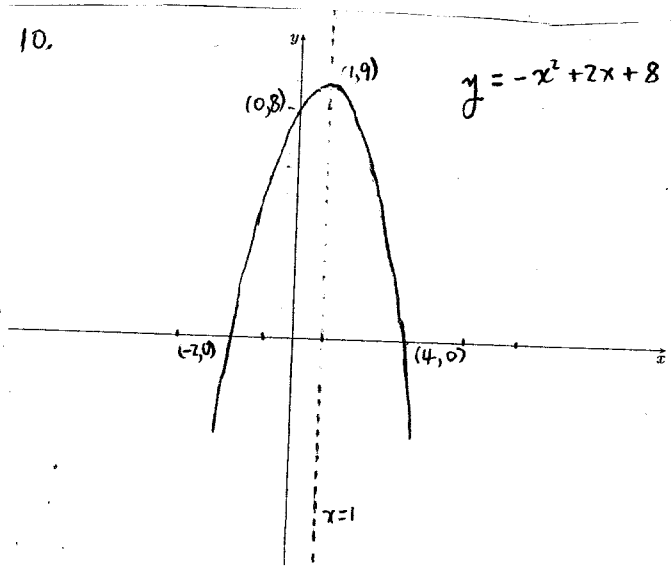
12. (a)

$y = \begin{cases} x^2-4 & \text{if } x \le 3 \\ x-6 & \text{if } x > 3 \end{cases}$



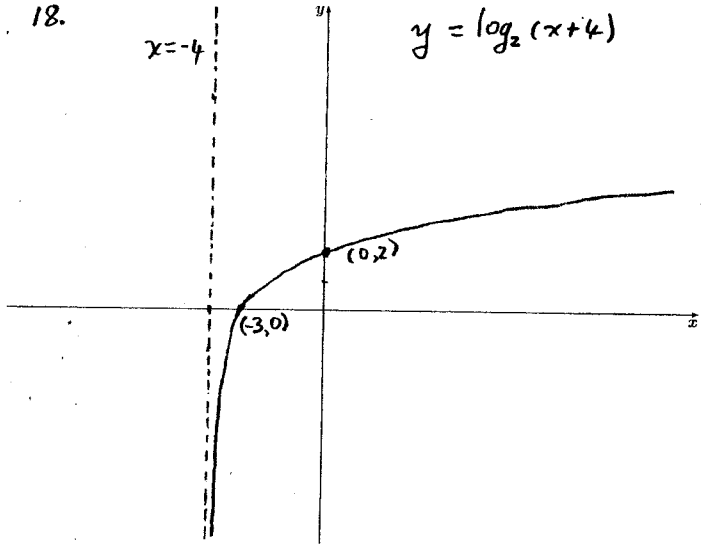
10.

$y = -x^2 + 2x + 8$



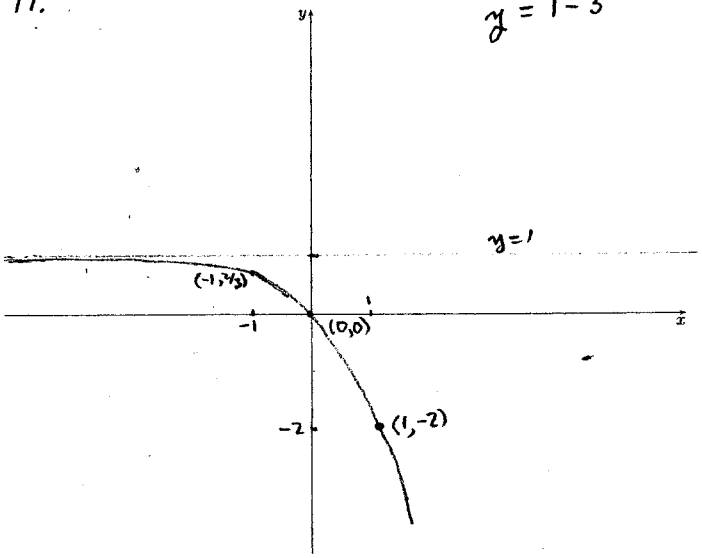
18.

$y = \log_2(x+4)$



19.

$y = 1 - 3^x$



26. (b)

$y = -3\sin\left(\frac{x}{4}\right)$