1. Use the Trapezoidal Rule with $n = 5$ to estimate the value of $\int_1^2 \frac{1}{x}\,dx$ (give your answer to 4 decimals).

2. Evaluate the following limits.

   (a) $\lim\limits_{x\to 0^+} x^{\sin x}$

   (b) $\lim\limits_{x\to \frac{\pi}{2}} \left(x - \frac{\pi}{2}\right) \cdot \tan(3x)$

   (c) $\lim\limits_{\theta\to 0} (\cot\theta - \csc\theta)$

3. Find y' for the following. **Do not simplify your answer.**

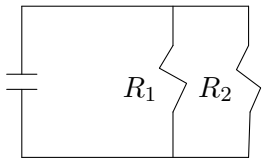
   (a) $y = x^2(\arcsin 2x)^4$

   (b) $y = \sec^{-1}(4x+1) + \sin(\ln x^2)$

   (c) $y = \arctan\sqrt{5x-6} + 4^x$

   (d) $y = \dfrac{\log_5 \sin x}{e^{\cos x}}$

4. If two resistors with resistance R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If R_1 andR_2 are increasing at rates of 0.40 Ω/s and 0.10 Ω/s, respectively, how fast is R changing when $R_1 = 50\ \Omega$ and $R_2 = 70\ \Omega$.



5. A poster is to have an area of 200 cm^2 with 2 cm margins at the bottom and sides and 4 cm margin at the top. What dimensions will give the largest printed area?

6. Given $f(x) = 2\sqrt{x} - x$ $\qquad f'(x) = \dfrac{-\sqrt{x}+1}{\sqrt{x}}$ $\qquad f''(x) = -\dfrac{\sqrt{x}}{2x^2}$.

   Find (if any):

   (a) The x and y intercept(s).
   (b) The vertical and horizontal asymptotes.
   (c) The critical numbers.
   (d) The inflection points.
   (e) Local (relative) extrema.

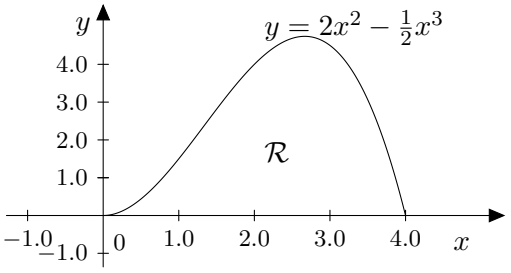
(f) Intervals of upward or downward concavity.

(g) Intervals on which f is increasing or decreasing.

(h) Sketch the graph of f.

7. Find the values of **c** such that the area of the region bounded by the parabolas $y_1 = x^2 - c^2$ and $y_2 = -x^2 + c^2$ is 576. **Hint**: $\int_{-c}^{c} f(x)\,dx = 2\int_{0}^{c} f(x)\,dx$

8. Let $\mathcal{R}$ be the region bounded by $y = 2x^2 - \frac{1}{2}x^3$, and $y = 0$. Set up, but **do not evaluate** the integral for the volume obtained by rotating the region $\mathcal{R}$ about the following:



(a) the y-axis

(b) the line $y = -1$

9. Solve the following differential equation for y.

$\sin 2x\,dx + y\sin x\,dy = \dfrac{1}{\csc x}\,dx$ with initial condition f(0)=2.

10. Integrate the following integrals.

(a) $\displaystyle\int \frac{6x^3 + 4x^2 - 90x - 64}{x^2 - 16}\,dx$

(b) $\displaystyle\int \frac{x}{\sqrt{x^2 - 4x}}\,dx$

(c) $\displaystyle\int \arctan\theta\,d\theta$

(d) $\displaystyle\int \tan^7\theta\sec^4\theta\,d\theta$

(e) $\int \cot^3\theta \sin^2\theta \, d\theta$

(f) $\int \cos\theta \cos^3(\sin\theta) \, d\theta$

(g) $\int \dfrac{e^x(e^x+4)}{e^x-2} \, dx$

11. Solve the following first order linear differential equation for y.
$y' + y\tan x = -\sin x$ with initial condition f(0)=5.

12. Find the first few non-zero terms of the Fourier series for the function

$$f(x) = \begin{cases} 1, & \text{if } -\pi \le x < 0 \\ x, & \text{if } 0 \le x < \pi \end{cases}$$

Hint: find $a_0, a_1, a_2, a_3, \ldots, b_1, b_2, \ldots$ and write the function expansion.

Answers

1. 0.6956

2. (a) 1
   (b) $-\dfrac{1}{3}$
   (c) 0

3. (a) $y' = 2x(\arcsin 2x)^4 + 4x^2(\arcsin 2x)^3 \cdot \dfrac{2}{\sqrt{1-4x^2}}$
   (b) $y' = \dfrac{4}{(4x+1)\sqrt{(4x+1)^2-1}} + \cos(\ln x^2) \cdot \dfrac{2}{x}$
   (c) $y' = \dfrac{1}{5(x-1)} \cdot \dfrac{5}{2\sqrt{5x-6}} + 4^x \ln 4$
   (d) $y' = \dfrac{e^{\cos x} \cdot \frac{\cos x}{\sin x \cdot \ln 5} - \log_5 \sin x \cdot e^{\cos x} \cdot -\sin x}{(e^{\cos x})^2}$

4. 0.15 Ω/s

5. 11.54cm by 17.32cm

6. (a) x-int $= 4$ and y-int $= 0$
   (b) V.A. None ; H.A. None
   (c) $x = 1$
   (d) None
   (e) Local Max. at $x = 1$
   (f) C.U. Nil.
   C.D. $(0, \infty)$

(g) Inc.$[0, 1[$; Dec.$]1, \infty[$

7. $c = 6$

8. (a) $V = 2\pi \displaystyle\int_0^4 x \left(2x^2 - \frac{1}{2}x^3\right) dx$

   (b) $V = \pi \displaystyle\int_0^4 \left(\left(2x^2 - \frac{1}{2}x^3\right)^2 - 1\right) dx$

9. $y = \sqrt{-4\sin x + 2x + 4}$

10. (a) $3x^2 + 4x + 3\ln|x^2 - 16| + c$

    (b) $\sqrt{x^2 - 4x} + 2\ln\left|\dfrac{x - 2 + \sqrt{x^2 - 4x}}{2}\right| + c$

    (c) $\theta \cdot \arctan\theta - \dfrac{1}{2}\ln|1 + \theta^2| + c$

    (d) $\dfrac{1}{10}\tan^{10}\theta + \dfrac{1}{8}\tan^8\theta + c$

    (e) $\ln|\sin\theta| - \dfrac{1}{2}\sin^2\theta + c$

    (f) $\sin(\sin\theta) - \dfrac{1}{3}\sin^3(\sin\theta) + c$

    (g) $e^x + 6\ln|e^x - 2| + c$

11. $y = -\ln|\sec x| \cdot \cos x + 5\cos x$

12. $f(x) = \dfrac{2 + \pi}{4} - \dfrac{2}{\pi}\cos x - \dfrac{2}{9\pi}\cos 3x - \cdots + \left(\dfrac{\pi - 2}{\pi}\right)\sin x - \dfrac{1}{2}\sin 2x$