

Question 1: (30 pts) Evaluate each of the following integrals, without the use of integration tables.

a) $\int \frac{t^4 + 3t - 5\sqrt{t} + t^2 6^t}{t^2} dt$ b) $\int (2x + 5)^2 \sin(x) dx$ c) $\int (x + 3) e^{x^2+6x} dx$

d) $\int \frac{x^4 + x^3 + x^2 + 3}{x^2 + x - 2} dx$ e) $\int \frac{3x + 43}{(x - 2)(x + 5)^2} dx$ f) $\int_0^{\pi/4} \sec^2(x) \sqrt{2 - \tan(x)} dx$

g) $\int_{-3}^0 \frac{x}{(x + 4)^{\frac{3}{2}}} dx$ h) $\int_1^e \frac{\ln(x)}{x^7} dx$

Question 2: (2 pts) If the marginal cost function for a product is $\frac{dC}{dx} = 10e^{-x/10}$, and if 10 items cost \$163.22, find the cost function $C(x)$.

Question 3: (4 pts) Given the functions $f(x) = 2 - x^2$ and $g(x) = x$

- Sketch the region R bounded by the graphs of $f(x)$ and $g(x)$.
- Find the area of the region R .

Question 4: (5 pts) Given the demand function $p_1(x) = \frac{20}{x + 1}$ and the supply function $p_2(x) = x + 2$

- Find the equilibrium point.
- Sketch and identify the regions representing the consumer and producer surpluses.
- Evaluate the producer surplus.

Question 5: (4 pts) Use the Trapezoidal rule to approximate $\int_{-2}^1 \frac{4}{1 + x^2} dx$. Use $n = 6$, and round your answer to four decimal places.

Question 6: (6 pts) Use the table of integrals to solve the following. In each case, state the formula number and justify its use.

a) $\int \frac{e^{2x}}{(2 + e^x)^2} dx$ b) $\int \frac{\sqrt{x^2 + 6x + 34}}{x + 3} dx$

Question 7: (2 pts) Determine whether $y = e^{x^2}$ is a solution to the differential equation $y'' - xy' = 2y$.

Question 8: (8 pts) Solve the following differential equations.

- $(x + 2) y' = 2y$, with $y = 5$ when $x = -1$ and $y > 0$.
- $x \frac{dy}{dx} = \frac{\ln(x) + 1}{y}$, with $y(1) = 3$ and $y > 0$.

Question 9: (4 pts) A rumor is started in a population of 1500. The rumor spreads at a rate that is proportional to the number of people who have not yet heard it. The rumor started with 10 people, and after 1 day 150 people have heard it.

- a) Find the function $N(t)$ for the number of people who have heard the rumor after t days.
b) How long will it take for the rumor to reach 1125 people?

Question 10: (6 pts) Evaluate the following limits:

a) $\lim_{x \rightarrow 0} \frac{x^2 + 3x + \cos(2x) - 1}{\sin(7x)}$ b) $\lim_{x \rightarrow +\infty} \frac{x}{\ln(x + e^x)}$

Question 11: (8 pts) Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

a) $\int_0^{\pi/2} \frac{\sin(x)}{1 - \cos(x)} dx$ b) $\int_5^{+\infty} \frac{x}{(x^2 - 9)^{\frac{3}{2}}} dx$

Question 12: (3 pts) Consider the sequence $\left\{ 2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}, \dots \right\}$

- a) Give the 5th term of the sequence. b) Find an expression for the n^{th} term of the sequence.

Question 13: (6 pts) Determine if the following sequences converge or diverge. If the sequence converges, find its limit.

a) $a_n = \frac{\sqrt{4n^2 + 1}}{n}$ b) $a_n = \frac{(n + 1)!}{(n + 2)!}$

Question 14: (6 pts) Determine if the following series converge or diverge. If the series converges, find its sum.

a) $\sum_{n=1}^{\infty} \frac{n^2 - 3n + 2}{5n^2 + 4}$ b) $\sum_{n=0}^{\infty} \left[\frac{3^{n+1}}{5^n} + \left(\frac{-2}{5} \right)^n \right]$

Question 15: (3 pts) A deposit of \$5 is made in an account at the beginning of every week, over a period of 20 years. The account earns an annual interest rate of 1.04%, compounded weekly. What will be the balance in the account at the end of the 20 years?

Question 16: (3 pts) Given the number $1.0\bar{3}$, express it using a geometric series. Find the sum of the geometric series to write the number as the ratio of two integers.

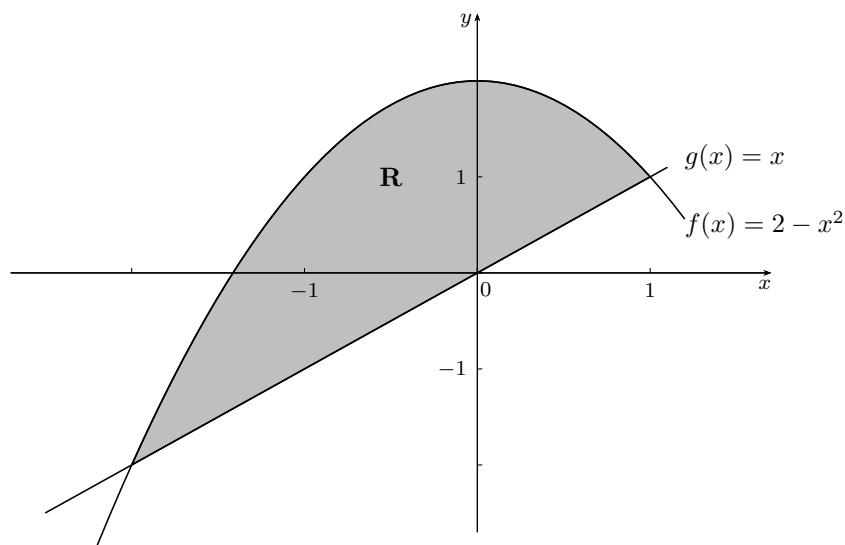
ANSWERS

1.) a) $\frac{1}{3}t^3 + 3 \ln |t| + \frac{10}{\sqrt{t}} + \frac{6^t}{\ln(6)} + C$ b) $-(2x + 5)^2 \cos(x) + (8x + 20) \sin(x) + 8 \cos(x) + C$

c) $\frac{1}{2}e^{x^2+6x} + C$ d) $\frac{x^3}{3} + 3x + 2 \ln |x - 1| - 5 \ln |x + 2| + C$ e) $\ln |x - 2| - \ln |x + 5| + \frac{4}{x + 5} + C$

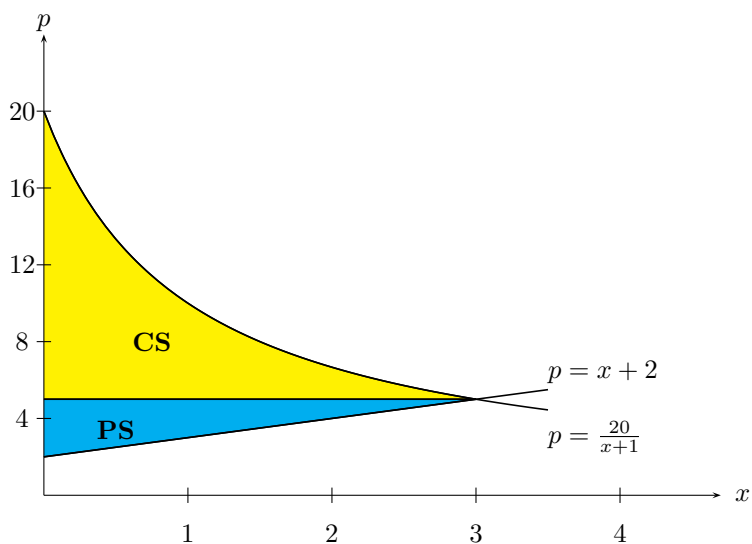
f) $\frac{4\sqrt{2}-2}{3}$ g) -2 h) $\frac{e^6-7}{36e^6}$ 2.) $C(x) = 200 - 100e^{-x/10}$

3.) a)



b) $\frac{9}{2}$ units² 4.) a) $E = (3, 5)$

b)



c) PS = \$4.50

- 5.) 7.5154 6.) a) $\ln|2 + e^x| + \frac{2}{2+e^x} + C$ b) $\sqrt{x^2 + 6x + 34} - 5 \ln \left| \frac{5 + \sqrt{x^2 + 6x + 25}}{x + 3} \right| + C$
- 7.) Not a solution 8.) a) $y = 5(x + 2)^2$ b) $y = \sqrt{(\ln(x) + 1)^2 + 8}$
- 9.) a) $N(t) = 1500 - 1490 \cdot \left(\frac{1350}{1490}\right)^t$ b) 14 days 10.) a) $\frac{3}{7}$ b) 1
- 11.) a) Diverges b) $\frac{1}{4}$ 12.) a) $\frac{26}{5}$ b) $\frac{n^2 + 1}{n}$ 13.) a) 2 b) 0
- 14.) a) Diverges b) $\frac{115}{14}$ 15.) \$5780.85 16.) $\frac{31}{30}$