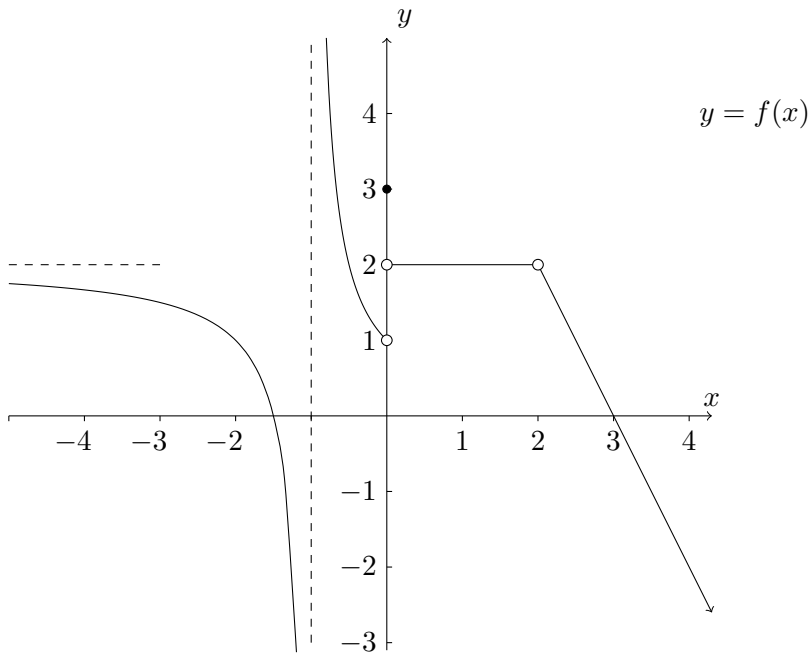


(Marks)

- (4) 1. Given the graph of f below, evaluate each of the following. Use ∞ , $-\infty$ or “does not exist” where appropriate.



- a) $\lim_{x \rightarrow 2} f(x) =$ b) $\lim_{x \rightarrow -1} f(x) =$ c) $f(0) =$
 d) $\lim_{x \rightarrow 0^-} f(x) =$ e) $\lim_{x \rightarrow -\infty} f(x) =$ f) $f(2) =$
 g) List the discontinuities of $f(x)$.

- (16) 2. Evaluate the following limits. Use ∞ , $-\infty$ or “does not exist” where appropriate.

a) $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{2x^2 - x - 3}$

b) $\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4}$

c) $\lim_{x \rightarrow 3} \frac{\frac{1}{x+1} - \frac{1}{4}}{x - 3}$

d) $\lim_{x \rightarrow -4} \frac{x + 5}{x^2 - 16}$

e) $\lim_{x \rightarrow -\infty} \frac{(x - 3)(3x^2 + 1)}{2x^3 - 5}$

(Marks)

$$f) \lim_{x \rightarrow 5^+} \frac{3|5-x|}{x^2 - 12x + 35}$$

- (3) 3. Find the point(s) of discontinuity for the following function. Justify using the definition of continuity.

$$f(x) = \begin{cases} \frac{7}{x^2 + 2x - 15} & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ x^2 - 5 & \text{if } x > 2 \end{cases}$$

- (3) 4. Find the value(s) for a and b such that the following function is continuous for all real numbers.

$$f(x) = \begin{cases} 5 & \text{if } x \leq 0 \\ ax - b & \text{if } 0 < x < 8 \\ 3 & \text{if } x \geq 8 \end{cases}$$

- (1) 5. **True or False:** If $f(c)$ is undefined then $\lim_{x \rightarrow c} f(x)$ does not exist. **Briefly** justify your answer.

- (1) 6. (a) State the limit definition of the derivative.

- (3) (b) Use the definition to find the derivative of $f(x) = \frac{1}{3-2x}$.

- (20) 7. Find the derivative for each of the following functions. Do not simplify.

$$a) y = 8x^3 - \frac{4}{x} + \frac{5}{\sqrt{(3x)}} + 4\pi^2$$

$$b) y = 3(2x + 5)^2(3x - e^{2x})^6$$

$$c) y = \log_2(\sqrt{x}) + 4\sqrt[3]{x^5} - 4 \cot(2x - 3)$$

$$d) y = \frac{\sin^2(4x)}{x^3 + 2x}$$

$$e) y = 2x^{x^2+1}$$

$$f) y = \ln \left[\left(\frac{(\sqrt{x-5}) e^{5x}}{(3x+1) \sec x} \right)^2 \right]$$

- (3) 8. Find the x -coordinates of the point(s) on the curve $y = x^3 e^{2x}$ that have horizontal tangents.

(Marks)

- (4) 9. Find $\frac{d^4y}{dx^4}$ given $y = 3^{2x+1} + \ln(\pi)x^3 - \sin x$.
- (5) 10. Let $x^2y^2 + y \ln(x) = 4x$.
- a) Find $\frac{dy}{dx}$
- b) Find the equation of the tangent line to the curve at $(1, 2)$
- (10) 11. Given $f(x) = \frac{2x^2 - 18}{x^2 - 4}$ with $f'(x) = \frac{20x}{(x^2 - 4)^2}$ and $f''(x) = \frac{-20(3x^2 + 4)}{(x^2 - 4)^3}$,
- (a) Find the x - and y - intercepts (if any).
- (b) Find the vertical and horizontal asymptotes (if any).
- (c) Give the intervals where $f(x)$ is increasing and decreasing, and the relative extrema (if any).
- (d) Give the intervals where $f(x)$ is concave up and concave down, and the points of inflection (if any).
- (e) Sketch a labelled graph of $f(x)$.
- (3) 12. Use the second derivative test to find all the relative (local) extrema of $f(x) = (x^2 - 9)^2$
- (3) 13. Find the absolute (global) extrema of $f(x) = x^3(x - 5)^2$ on the interval $[1, 4]$.
- (5) 14. A rectangular storage container with an open top is to have a volume of 32 m^3 . The length of the base is four times its width. Material for the base costs \$5 per square meter. Material for the sides costs \$4 per square meter. Find the cost of materials for the cheapest such container.
15. Let the revenue function of a product be given by $R = -0.003x^2 + 5x$ and the average cost function be
- (5) given by $\bar{C}(x) = \frac{300}{x} + 1.1$, where $0 \leq x \leq 800$.
- a) Determine the production level that will maximize profit.
- b) At this production level, what comparison can we make regarding marginal revenue and marginal cost?
- (5) 16. A sugar shack has been trying to attract more customers and introduced the following pricing scheme: Groups up to ten pay \$20 per ticket. For each additional ticket, all group members receive a \$0.20 discount. What size group would maximize the revenue of this sugar shack?
- (6) 17. The demand curve for a product is given by $x = 3000 - 250p^2$ where x is the production level and p is the unit price in dollars.

(Marks)

- a) Determine the price elasticity of demand function η .
 b) What is the price elasticity of demand when the price is \$3? Is demand elastic, inelastic or unit elastic?
 c) At the price of \$3, if the price decreases by 3%, how is demand affected?
 d) Determine the price that would maximize the revenue.

ANSWERS

1.a) 2 b) DNE c) 3 d) 1 e) 2 f) DNE g) Discontinuities at $x = -1, x = 0, x = 2$

2.a) $-\frac{1}{5}$ b) $\frac{1}{6}$ c) $-\frac{1}{16}$ d) DNE e) $\frac{3}{2}$ f) $-\frac{3}{2}$ 3) Discts at $x = -5, x = 2$

4) $a = \frac{-1}{4}, b = -5$ 5) False 6) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ b) $\frac{2}{(3-2x)^2}$

7.a) $y' = 24x^2 + \frac{4}{x^2} - \frac{5}{2\sqrt{3}}x^{-3/2}$ b) $y' = 3(2)(2x+5)(2)(3x - e^{2x})^6 + 3(2x+5)^2(6)(3x - e^{2x})^5(3 - 2e^{2x})$

c) $y' = \frac{1}{2x \ln 2} + 4 \left(\frac{3}{5}x^{2/3} + 4 \csc^2(2x-3)(2) \right)$ d) $y' = \frac{2 \sin(4x) \cos(4x)(4)(x^3 + 2x) - \sin^2(4x)(3x^2 + 2)}{(x^3 + 2x)^2}$

e) $y' = 2x^{x^2+1} \left(2x \ln x + \frac{x^2+1}{x} \right)$ f) $y' = 2 \left(\frac{1}{2(x-5)} + 5 - \frac{3}{3x+1} - \tan x \right)$

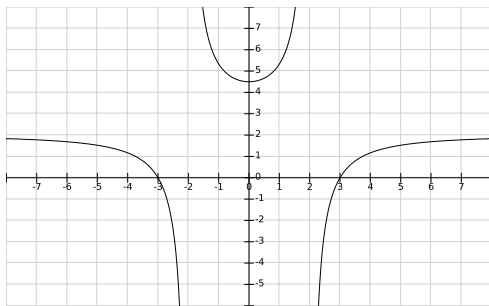
8) $x = 0, \frac{-3}{2}$ 9) $y' = 3^{2x+1}(2 \ln 3)^4 - \sin x$

10.a) $y' = \frac{-(2x^2y^2 - 4x + y)}{(2x^3y + x \ln(x))}$

b) $y = \frac{-3}{2}x + \frac{7}{2}$ 11) Intercepts $(3, 0), (-3, 0), (0, 4.5)$ b) VA: $x = -2, x = 2$ HA: $y = 2$

c) INC: $(0, 3) \cup (2, \infty)$ DEC: $(-\infty, -2) \cup (-2, 0)$ Local Max: NONE, local Min: $(0, 4.5)$

d) CU: $(-2, 2)$ CD: $(-\infty, 2) \cup (2, \infty)$, no inflection points.



12) Local Max: $(0, 81)$, Local Mins: $(-3, 0), (3, 0)$

13) Abs Max of 108 at $x = 3$, Abs Min of 16 at $x = 1$ 14) $2\text{m} \times 8\text{m} \times 2\text{m}$

15.a) A production level of 650. b) They are equal ($R' = C'$) 16) A group of 55 people

17.a) Taking η as a function of p , $\eta = \frac{p}{xp'} = \frac{-500p^2}{3000 - 250p^2}$ b) $\eta(3) = -6$. Elastic

c) Demand increases by 18% d) At a price of \$2