1. (5 points) (a) Find the exact value of $\cos\left(\tan^{-1}\left(\frac{3}{2x}\right)\right)$

(b) Use your answer to part (a) to evaluate $\int \cos\left(\tan^{-1}\left(\frac{3}{2x}\right)\right)\, dx$

2. (25 points) Evaluate the integrals

(a) $\int \frac{11x^2 - 14x + 8}{(2x-1)(x^2+1)} dx$

(b) $\int_0^{\frac{1}{4}} \frac{\arccos(2x)}{\sqrt{1-4x^2}}\, dx$

(c) $\int x^5 \cos(x^2)\, dx$

(d) $\int \frac{\sin^3(5x)}{\cos(5x)}\, dx$

(e) $\int \frac{\sqrt{1-x^2}}{x^4}\, dx$

3. (10 points) Evaluate the improper integrals.

(a) $\int_5^{\infty} \frac{1}{x^2 - 10x + 29}\, dx$

(b) $\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{\sqrt{\tan x}}\, dx$

4. (9 points) Evaluate the limits.

(a) $\lim_{x\to\frac{\pi}{6}} \sec(3x)\sin\left(x - \frac{\pi}{6}\right)$

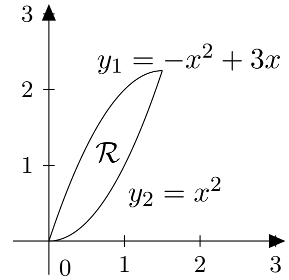
(b) $\lim_{x\to e^-} [\ln x]^{\frac{2}{1-\ln x}}$

(c) $\lim_{x\to 0} \frac{\arctan(2x)}{\arctan(5x)}$

5. (5 points) Find the area between $y_1 = \sqrt{x+1}$ and $y_2 = \frac{x+1}{2}$

6. (4 points) Let $\mathcal{R}$ be the region bounded by $y_1 = -x^2 + 3x$ and $y_2 = x^2$. Set up, but **do not evaluate** the integral for the volume obtained by rotating the region $\mathcal{R}$ about the following:

(a) the $y-$axis



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(b) the line $y = -1$

7. (4 points) Find the length of the curve $y = 2\ln\left(\cos \frac{1}{2}x\right)$ from $\frac{\pi}{3} \leq x \leq \frac{\pi}{2}$

8. (4 points) Solve the differential equation: $x^2y' + 2xy = 3x$, given $y(1) = 1$ and $x > 0$. Express y as a function of x.

9. (2 points) Determine if the sequence $a_n = \frac{4}{9^n} + 3\arctan(\ln(n^2))$ converges or diverges. If it converges, find its limit.

10. (12 points) Determine whether each of the following series converges or diverges. State the test used and justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+n}{n2^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+1}\right)^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{7}{4^n}\right)$

11. (6 points) Determine whether each of the following series is absolutely convergent, conditionally convergent or divergent. Justify your answer.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\ln(4n)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2 5^n}{(2n)!}$

12. (5 points) Determine whether each of the following series converges or diverges. If it converges find the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \left[\arccos\left(\frac{1}{n+1}\right) - \arccos\left(\frac{1}{n+2}\right)\right]$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n}$

13. (2 points) Suppose that $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -5$ and diverges when $x = 7$. What can be said about the convergence or divergence of the following series? Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty} c_n(-4)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} c_n(5)^n$

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(c) $\sum_{n=0}^{\infty} c_n$

(d) $\sum_{n=0}^{\infty} c_n(-8)^n$

14. (3 points) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n\, 3^{2n}(x-2)^n}{4n+1}$

15. (4 points) Let $f(x) = \frac{1}{2-x}$

(a) Write the first five nonzero terms of the Taylor series for $f(x)$ centered at $a = 5$.

(b) Find a formula for the n^{th} term of the series, and express the series in sigma notation.