

# Answers to final exam NYB Fall 2013

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1. a)  $\frac{2|x|}{\sqrt{4x^2+9}}, x \neq 0$       b)  $\frac{|x|\sqrt{4x^2+9}}{2x} + c$       2. a)  $\frac{3}{2}\ln|2x-1| + 2\ln(x^2+1) - 5\arctan x + c$

b)  $-\frac{1}{4}\arccos^2(2x) \Big|_0^{1/4} = \frac{5\pi^2}{144}$       c)  $\frac{1}{2}\left[x^4 \sin(x^2) + 2x^2 \cos(x^2) - 2\sin(x^2)\right] + c$

d)  $-\frac{1}{5}\left[\ln|\cos(5x)| - \frac{\cos^2(5x)}{2}\right] + c$       e)  $-\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + c$

3. a)  $\lim_{x \rightarrow \infty} \frac{1}{2}\arctan\left(\frac{x-5}{2}\right) \Big|_5^x = \frac{\pi}{4}$       b)  $\lim_{x \rightarrow 0^+} 2\sqrt{\tan x} \Big|_x^{\pi/4} = 2$

4. a)  $-\frac{1}{3}$       b)  $e^{-2}$       c)  $\frac{2}{5}$

5.  $\frac{4}{3}\text{units}^2$       6. a)  $2\pi \int_0^{3/2} x(-x^2+3x-x^2)dx$       b)  $\pi \int_0^{3/2} (1+3x-x^2)^2 - (1+x^2)^2 dx$

7.  $2\ln\left(\frac{(2+\sqrt{2})\sqrt{3}}{3\sqrt{2}}\right)$       8.  $y = \frac{3}{2} - \frac{1}{2x^2}$       9. *conv. to*  $\frac{3\pi}{2}$

10. a) *conv. by comp.test or integral test*      b) *conv. by ratio.test*  
 c) *conv. by Root.test*      d) *Div. by Divergencetest*

11. a) *conv. by Alternating series test and div. by Limit Comparaison test. So it is conditionally convergent.*  
 b) *Absolutely Conv. By Ratio test*

12. a) *conv. telescoping*  $sum = -\frac{\pi}{6}$       b) *Geometric conv.*  $sum = \frac{1}{3}$

13. a) *Conv.*      b) *we can not say anything about this series as x=5 might be the other end point of interval of convergence*      c) *Conv.*      d) *Div.*

14.  $R = \frac{1}{9}$  and the interval  $\frac{17}{9} < x \leq \frac{19}{9}$

15. a)  $-\frac{1}{3} + \frac{1(x-5)}{9} - \frac{2(x-5)^2}{27 \cdot 2!} + \frac{6(x-5)^3}{81 \cdot 3!} - \frac{24(x-5)^4}{243 \cdot 4!}$

b)  $-\frac{1}{3} + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-5)^n}{3^{n+1}}$