

Part I (One mark each)

Answer true if the statement is always true. Answer false otherwise.

1. If  $X$  is a binomial random variable, then  $E(X^2) = (np)^2 + np(1-p)$
2. Increasing the value of  $\alpha$  (all else remaining unchanged) will increase the width of the confidence interval for a particular parameter.
3. For a normal distribution, a value that is one standard deviation above the mean is approximately the 84th percentile.
4. If  $A$  and  $B$  are two events defined on a common sample space  $S$ , then  $P(A \cap B) + P(A \cap B^c) = P(A)$ .
5. For the hypotheses  $H_0 : \frac{\sigma_1^2}{\sigma_2^2} \geq 1$  vs.  $H_a : \frac{\sigma_1^2}{\sigma_2^2} < 1$  a rejection of the null hypothesis would indicate that population 2 is more variable than population 1.

Part II (Three marks each)

1.  $A$  and  $B$  are events defined on a common sample space. If  $P(A \cap B^c) = 0.1$ ,  $P(A^c \cap B) = 0.2$  and  $P(A \cap B) = k$ , find the value(s) of  $k$  for which  $A$  and  $B$  are independent. Hint: Use a Venn diagram.
2. For the data

x	3	7	k
y	4	12	10

Find  $k$  so that the value of the linear correlation coefficient  $r$  is exactly 1.

3. A carton of eight light bulbs contains two that are defective. Let  $x$ =number of defectives when two bulbs are randomly selected (one at a time, without replacement) from this carton. What is the probability that at least one of the bulbs selected is defective?
4. If 40% of all students who enter a certain university drop out during or at the end of the first year, what is the approximate probability that more than 600 of this year's entering class of 1600 will drop out during or at the end of their first year?
5. A 98% confidence interval for  $\mu$  is computed to be:  $32 < \mu < 38$ . If  $\sigma^2 = 66.6$ , find the sample size,  $n$ . (We know the sample size is large.)
6. What is the minimum sample size required to estimate the proportion of success  $p$  in a binomial population to within 3% with 80% confidence?
7. Given  $H_0 : p_1 - p_2 = 0$  vs.  $H_a : p_1 - p_2 \neq 0$ , find the absolute value of the test statistic whose p-value is 0.0456.
8. If  $X$  is a Poisson random variable with  $\lambda = 6$ , find  $E(y)$  where  $y = 3x + 7$ .
9. If  $X$  is a Binomial random variable with  $n = 6$  and  $p = 0.5$ , find the probability that  $x$  takes on a value within one standard deviation of the mean.

Part III (6 marks each)

1. The cholesterol of a sample of 10 men is measured and a hypothesis test is performed at the  $\alpha = 0.05$  level of significance to test if the mean cholesterol level in men is greater than 225. It is known that cholesterol is normally distributed with standard deviation 30. If the actual mean cholesterol level is 270, find the probability of Type II error.
2. Four cards are to be randomly dealt from a standard deck of 52 cards (no jokers).
  - a) How many different hands of four cards are possible?
  - b) How many different hands of four cards are possible if exactly one of the cards dealt is a heart?

- c) How many different hands of four cards are possible if **at least** one of the cards dealt is a heart?
- d) How many different hands of four cards are possible if exactly one of the cards drawn is a spade and the remaining cards are not queens?
- e) What is the probability that all four cards are from the same suit?
3. A company that manufactures shoes has three factories. Factory 1 produces 25% of the company's shoes and Factory 2 produces 60%. One percent of the shoes produced by Factory 1 are mislabeled, 0.5% of the shoes produced by Factory 2 are mislabeled and 2% of those produced by Factory 3 are mislabeled. If you purchase a pair of shoes manufactured by this company and found it to be mislabeled, what is the probability that it was produced by Factory 3?
4. The I.Q. scores of a population are normally distributed with a mean of 100 and a standard deviation of 12.
- a) What is the probability that a person chosen at random from this population will have an I.Q. score less than 90?
- b) What is the probability that the mean I.Q. score of a random sample of 9 people from this population will be less than 90?
5. Determine the p-value associated with each of the following hypothesis tests:
- a)  $H_0 : p_1 = 0.3$  vs.  $H_a : p_1 \neq 0.3$ ,  $n=64$ ,  $x=24$
- b)  $H_0 : \sigma^2 = 4$  vs.  $H_a : \sigma^2 > 4$ ,  $n=17$ ,  $s^2=8$
- c)  $H_0 : \mu = 20$  vs.  $H_a : \mu > 20$   $n=36$ ,  $s=5$ ,  $\bar{x} = 18.5$
6. The diastolic blood pressure for nine randomly selected patients was determined using two techniques: the standard method used by medical personnel and a method using an electronic device with a digital readout.

The results are:

Patient	1	2	3	4	5	6	7	8	9
Std method	72	80	88	65	75	96	79	75	62
Digital method	70	76	87	64	72	95	80	70	61

Assuming that diastolic blood pressure is a normally distributed random variable, do these data provide sufficient evidence that the mean of the digital method is lower than that of the standard method? (Use  $\alpha = 0.05$ .)

7. In preparation for contract renewal negotiations, the teachers' union surveyed its members to see whether they preferred a large increase in retirement benefits or a smaller increase in salary. In a sample of 1000 male teachers, 750 were in favour of increased retirement benefits. Of 500 female teachers, 410 favoured an increase in retirement benefits. Test at the  $\alpha = 0.05$  level of significance the hypothesis that there is no difference in the proportions of male and female teachers who are in favour of increased retirement benefits. Please test using the p-value method.

The following independent samples are to be used to answer problems 9 and 10. Each sample is taken from a normal population.

Sample 1	72	29	62	60	68	59	61	73	38	
Sample 2	75	43	63	61	72	73	82	47	63	43

8. a) At the 5% level of significance, can you conclude that the mean of population 2 is different than 55? Use the p-value approach.  
 b) Using sample 1, construct a 99% confidence interval for the mean of population 1.
9. a) At the 5% level of significance, can you conclude that population 1 is more variable than population 2? Use the classical approach.  
 b) Do these data provide sufficient evidence ( $\alpha = 0.02$ ) of a difference in population means?
10. You want to test whether attending class influences how students perform on an exam. Students from class of 54 are classified as follows:

	Pass	Fail
Attended	25	6
Skipped	8	15

Is there relationship between attendance and student success? Test using the classical approach and  $\alpha = 0.005$ .

Part IV (8 marks)

Given that X is a continuous random variable with probability density function

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ kx & \text{if } 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- a) Find  $k$ .  
 b) Compute  $P(1 \leq x \leq 2)$   
 c) Find  $E(x-1)$ .  
 d) Find the median