(Marks)

(2) 1. Evaluate the expressions:
(a) $6! - 4!$
(b) ${}_7P_3$

(2) 2. In a small gallery, there are seven places on the wall where large paintings can be hung. In how many ways can the gallery be filled with art if there are nine different paintings that can be hung?

(2) 3. The corner frozen yogurt shop sells 9 different flavors of frozen yogurt. Jeremy got a promotional card that allows him to stop by for a bowl of three scoops of frozen yogurt in the flavors of his choice. In how many ways could Jeremy order a three-scoop bowls of frozen yogurt if he is allowed to repeat flavors.

(4) 4. Let $A = \{1, 3, 7, 9\}$.
(a) How many subsets of A have at least 3 elements?
(b) Write all subsets of A with exactly 2 elements.
(c) What is the number of subsets of A?
(d) What is the number of proper subsets of A?

(4) 5. Consider the sets $A = \{x | \, x \text{ is a multiple of } 3\}$, $B = \{x | \, x \text{ is even}\}$, and the universal set U as the set of all integers from 1 to 20.
(a) Find $A \cap B$.
(b) Find $A \cap \overline{B}$.
(c) Are the sets $A \cap B$ and $A \cap \overline{B}$ equal? Explain.
(d) Are the sets $A \cap B$ and $A \cap \overline{B}$ equivalent? Explain.

(2) 6. Draw a Venn diagram and show by hatching the following set: $\overline{(A \cup B) \cap C}$

(2) 7. Name two properties of sets that allow us to determine that $A \cup \overline{A} \cap B = B$ for any sets A and B.

(1) 8. What is a tautology?

(3) 9. Associate each property listed with one of the following property names: *associative, closure, commutative, complement, contradiction, DeMorgan, distributive, double complement, double negation, idempotent, identity, property of 1 (or 0), tautology,*
and say whether it is a set property, a network property, or a logic property.

a) $(p \vee q) \leftrightarrow (q \vee p)$ _____

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ _____

c) $\overline{\overline{A}} = A$ _____

(2) 10. State both identity properties for logic.

(6) 11. Provide a truth table for each of the following statements. Determine whether the statement is a tautology, a contradiction or neither.
(a) $(\sim p \leftrightarrow q) \rightarrow (p \veebar q)$
(b) $(p \vee q) \barwedge (p \rightarrow r)$

(4) 12. If p represents a simple statement "*I eat breakfast.*" and q represents a simple statement "*I will be hungry.*"
(a) What expression in English is represented by $(\sim p \rightarrow q)$?
(b) What expression in English is represented by $(\sim p \wedge \sim q)$?
(c) Use truth tables to determine whether $[\sim (\sim p \rightarrow q)]$ is equivalent to $(\sim p \wedge \sim q)$.

(4) 13. Determine whether the given argument is valid:

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H: If you train at the gym, then you will get stronger.
You do not train at the gym.

C: You will not get stronger.

(3) 14. Use a Venn diagram to determine the validity of the argument.

H: Some redheads are Irish.
Nobody in class is a redhead.

C: Nobody in class is Irish.

(4) 15. Consider the quote *If you build it, then they will come.*

a) Write the converse, the inverse and the contrapositive of the implication above.
b) Say which among the four statements are equivalent.

(8) 16. For the given expression $(A+B)(\overline{A}+\overline{B})+A(\overline{B}+A)$.
(a) Construct the Boolean table.
(b) Draw the network.
(c) From all possible combinations of positions of the switches(A and B both open, A and B both closed, A open and B closed or A closed and B open) list the ones that would allow electricity to flow through the network.
(d) Simplify the expression as much as possible.

(3) 17. Simplify the expression $(A+B)(C\overline{A}+C\overline{C}+BC)$ justifying each step using properties of networks.

(7) 18. Find the intersection of the following pairs of lines, if it exists. Support your answers by graphing the lines.
(a) $2x+4y=-8$ and $6x+12y=24$
(b)$4x+6y=5$ and $-x+2y=-3$
(c) Which of the two systems above is inconsistent. Explain.

(10) 19. Given: $A=\begin{bmatrix} 1 & 5 \\ -2 & -6 \end{bmatrix}$ $B=\begin{bmatrix} 4 & 1 \\ 0 & 2 \\ 3 & -1 \end{bmatrix}$ $C=\begin{bmatrix} 2 & 3 & -2 \\ 1 & 1 & 5 \end{bmatrix}$

$D=\begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & -1 \\ -1 & 1 & 2 \end{bmatrix}$ $I_3=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

find each of the following, if possible. If an operation is not possible, say why.

a) $2DD^{-1}-3I_3$
b) CB
c) A^2
d) $B+I_3$
e) I_3C
f) C^T+B
g) DB

(6) 20. Given $A=\begin{bmatrix} 3 & 3 & 4 \\ 1 & -2 & 0 \\ -3 & -1 & -3 \end{bmatrix}$ find A^{-1} using elementary row operations.

(3) 21. Given $A=\begin{bmatrix} 1 & -4 \\ 2 & -8 \end{bmatrix}$, explain why A^{-1} does not exist.

(Marks)

(4) 22. Consider the system of linear equations

$$\begin{array}{rcrcrcr} x & - & 2y & - & 3z & = & 6 \\ -6x & + & 13y & + & 19z & = & 10 \\ -2x & + & 5y & + & 8z & = & -1 \end{array}$$

a) Write the system in matrix form $AX = B$.

b) Verify that the inverse matrix of A is $\begin{bmatrix} 9 & 1 & 1 \\ 10 & 2 & -1 \\ -4 & -1 & 1 \end{bmatrix}$

c) Use A^{-1} to solve given system of linear equations.

(1) 23. What is the solution of the system $AX = B$,
if the reduced form of augmented matrix is $\left[\begin{array}{ccc|c} 1 & -3 & 0 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

(6) 24. Solve, if possible, the following system using matrices and row reduction.

$$\begin{array}{rcrcrcr} 2x & + & 3y & - & 5z & = & -6 \\ x & + & y & + & z & = & 3 \\ -4x & - & 4y & + & 3z & = & -5 \end{array}$$

(5) 25. Solve, if possible, the following system using matrices and row reduction.

$$\begin{array}{rcrcrcr} 2x & + & 3y & + & 4z & = & 7 \\ x & + & 2y & - & 5z & = & 3 \\ 7x & + & 11y & + & 7z & = & 3 \end{array}$$

(3) 26. Use mathematical induction to prove that the following statement holds for all positive integers n:

$$2 + 7 + 12 + \cdots + (5n - 3) = \frac{n(5n-1)}{2}$$

ANSWERS:

1. (a) 696 (b) 210
2. 181440
3. 165
4. (a) 5, (b) $\{1,3\},\{1,7\},\{1,9\},\{3,7\},\{3,9\},\{7,9\}$ (c) 16, (d) 15
5. (a) $\{6,12,18\}$, (b)$\{3,9,15\}$ (c) No, they don't have the same elements.
   (d) Yes, they have the same number of elements.
6. Solution is everything except intersection of A with C, and intersection of B with C.
7. Complement and identity
8. Logical statement which is true in every case.
9. (a) Commutative (logic) (b) Distributive (sets) (c) Double complement (networks)
10. $(t \wedge p) \leftrightarrow p$ and $(c \vee p) \leftrightarrow p$
11. (a) Tautology (b) Contradiction

(Marks)

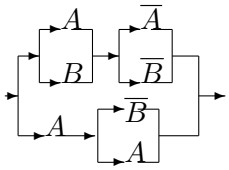
12. (a) "*If I don't eat breakfast, then I will be hungry.*"
(b) "*I don't eat breakfast and I won't be hungry.*"
(c) Equivalent

13. Invalid argument

14. Invalid argument

15. (a) Converse: If they will come, then you build it.
Inverse: If you don't build it, then they will not come.
Contrapositive: If they will not come, then you will not build it.
(b) The original statement is equivalent to the contrapositive. The converse statement is equivalent to the inverse.

16. (a) It is 0 when A and B are both 0, and 1 in the other three cases.

(b) 

(c) A, B both closed; A open and B closed; A closed and B open. (d)$A + B\overline{A}$

17. BC

18. (a) Parallel lines, no solution. (b) $x = 2, y = -1/2$ (c) System with no solution from (a).

19. (a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 10 \\ 19 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} -9 & -25 \\ 10 & 26 \end{bmatrix}$ (d) Undefined
(e) Undefined (f) $\begin{bmatrix} 6 & 2 \\ 3 & 3 \\ 1 & 4 \end{bmatrix}$ (g) $\begin{bmatrix} 13 & -6 \\ 9 & 8 \\ 2 & -1 \end{bmatrix}$

20. $\begin{bmatrix} -6 & -5 & -8 \\ -3 & -3 & -4 \\ 7 & 6 & 9 \end{bmatrix}$

21. No inverse, A can not be reduced to I.

22. a) $\begin{bmatrix} 1 & -2 & -3 \\ -6 & 13 & 19 \\ -2 & 5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ -1 \end{bmatrix}$ b) $AA^{-1} = I$ (c) $X = A^{-1}B = \begin{bmatrix} 63 \\ 81 \\ -35 \end{bmatrix}$

23. $\begin{aligned} x &= 1 + 3t \\ y &= t \\ z &= -2 \end{aligned}$

24. $\begin{aligned} x &= 7 \\ y &= -5 \\ z &= 1 \end{aligned}$

25. System inconsistent (no solution).

26. $P_1 : 2 = \frac{2(5\cdot 2-1)}{2}$ true
Assuming that P_k is true, P_{k+1} : LHS $= \frac{k(5k-1)}{2} + 5(k+1) - 1 = \frac{(k+1)(5k+4)}{2} =$ RHS.