

1. (2 points) You are making your schedule for next semester. You need to choose one sociology course, one psychology, and one physical education. Suppose in the course calendar, there are three available sociology courses, two available psychology courses and five physical education courses. How many different schedules can you make with these choices?
2. A group intervention is scheduled for Bob, who has a drug abuse problem. In addition to Bob, five more people will be invited to participate.
 - (a) (2 points) Suppose that the support group needs to consist of two friends and three family members. Suppose that Bob has a total of 6 friends, and 8 family members close to him. How many possible support groups can be formed?
 - (b) (2 points) Now assume the invitees have been chosen. At the time of the intervention, the 6 people invited (including Bob) will arrive separately at the location, at slightly different times. How many different arrival orders are possible?
3. (3 points) As you are driving home from work, you need to pass two intersections with traffic lights. Suppose you have a 40% chance of stopping at the first intersection (because of a red light), and a 60% chance of stopping at the second intersection. Moreover, if you had to stop at the first intersection, there is a 75% chance that you'll also need to stop at the second intersection.
Find the probability of stopping at both intersections, in other words getting a red light at both intersections.

4. A group of 150 people were classified by learning preference (visual or auditory learners), and by their math skills (good, average or weak). Suppose we pick one person at random from this group.

		Learning Preference		
		<i>visual</i>	<i>auditive</i>	Total
Math Skills	<i>good</i>	25	5	30
	<i>average</i>	62	40	102
	<i>weak</i>	10	8	18
	Total	97	53	150

- (a) (2 points) Find the probability that this person is a visual learner.
- (b) (2 points) Find the probability that this person is an auditory learner and has good math skills.
- (c) (2 points) Find the probability that this person has good math skills given that he's visual.
- (d) (3 points) Are the events "good at math" and "is a visual learner" independent? Explain.
5. (5 points) Suppose that 10% of the population are consulting a psychiatrist. Among those who are consulting, 28% consult because they suffer from depression. Among those who are not consulting a psychiatrist, 20% suffer from depression. Suppose one of your close friends told you she feels very depressed. What is the probability that she's consulting a psychiatrist?
6. Based on past experience, suppose that 9% of John Abbott students schedule their exams with the Student Access Centre because of their special needs. Consider a teacher having a small group of 15 random students.
 - (a) (2 points) Find the probability that exactly 2 students in this group will schedule their exams with the Student Access Centre.
 - (b) (4 points) Find the probability that 3 or more students in this group will schedule their exams with the Student Access Centre.

7. Let the random variable X denote the number of cats owned by a random citizen of Springfield. Suppose the probability distribution for X is given by:

x	0	1	2	3	4	15
$P(x)$	0.54	0.19	0.17	0.07	0.02	0.01

- (a) (3 points) Calculate the expected value (mean) of X .
- (b) (3 points) Calculate the standard deviation of X .
8. The age (in months) at which a child learns how to count to ten has a normal distribution with a mean of 24 months and a standard deviation of 6 months.
- (a) (3 points) If we consider one child taken at random, what is the probability that he will learn counting to ten before he's 19 months old?
- (b) (4 points) What is the probability that a child learns to count to ten between the age of 24 months and 37 months?
- (c) (4 points) Now suppose we consider a group of 15 children. What is the probability that the average learning age to count to ten is between 19 months and 29 months?
9. It was found that about 20% of people dream in black and white only. Consider running a study with a random sample of 200 people. Use a normal approximation to the binomial distribution in order to approximate the probability that:
- (a) (4 points) 45 or less people dream in black and white in this group,
- (b) (5 points) between 45 and 55 people (inclusively) dream in black and white in this group.
10. (4 points) By running a survey with 91 students, the sample average time spent procrastinating online was found to be 0.26 hours per day, with a sample standard deviation of 0.95 hours per day. Construct an 85% confidence interval for the average time spent procrastinating online for all students.
Interpret your answer in the context of this problem.
11. (4 points) A researcher wants to estimate, by using a 98% confidence interval, the average time psychiatrists spend with their patients per week. How big a sample of psychiatrists should he consider in order to estimate this average within 2 hours? Assume the population standard deviation for the time spent with patients is 6 hours per week.
12. In the United States, it is claimed that 20% of psychiatrists are board certified. In an attempt to disprove this claim, you gather a random sample of 55 psychiatrists, and find that 15 of them are board certified.
- (a) (5 points) Does this data indicate that the actual proportion of certified psychiatrist is different from 0.20? Use a 10% significance level and the P -value approach.
- (b) (2 points) Are there conditions that need to be verified before using the test in part (a)?
If so, verify whether these conditions are met in this problem.
- (c) (2 points) Interpret your conclusion in the context of this problem.

- 13.** A study using 30 female and 51 male psychiatrists has obtained an average salary of 151,000\$ for females (with sample standard deviation of 40,000\$) and a sample average salary of 182,000\$ for male psychiatrists (with sample standard deviation of 30,000\$).
- (5 points) Test the claim that men earn significantly more money than women in the psychiatrist profession. Use a 1% significance level and the rejection region approach.
 - (2 points) Estimate the P -value.
 - (2 points) Interpret your conclusion in the context of this problem.
- 14.** (4 points) Consider a researcher interested in whether people who majored in psychology are more or less likely than physics majors to solve a problem that involves statistical reasoning. Random samples of 100 psychology majors and 110 physics majors are taken and each person is given a chance to solve the problem. Of the 100 psychology majors, 65 solved the problem; of the 110 physics majors only 45 solved it. Construct a 99% confidence interval for the difference in the proportions of people between psychology and physics majors who can solve the statistical reasoning problem.
- 15.** A random sample of 5 students who feel extremely stressed and anxious before the exam period are meeting with the school counsellor (psychologist by training). The heart rate (beats per minute) of these students is measured before, and immediately after meeting with the counsellor. The data is shown in the table below.

Student	1	2	3	4	5
Heartbeat before meeting	100	98	115	85	104
Heartbeat after meeting	97	92	113	87	81

- (7 points) Based on this data, test the hypothesis that the school counsellor is effective at reducing the stress and anxiety of students during the exam period. Use a 5% significance level and the rejection region approach.
 - (2 points) Estimate the P -value.
- 16.** (6 points) A paper summarizes the results of an extensive study in the United States concerning psychiatrists. In particular, about the number of hours spent with patients per week, this paper claims the following distribution among psychiatrists in the U.S.:

Number of hours per week	less than 30	between 30-40	between 41-45	more than 45
Percentage of psychiatrists	30%	40%	10%	20%

After doing a similar study in Canada using 80 psychiatrists, you find the following data:

Number of hours per week	less than 30	between 30-40	between 41-45	more than 45
Number of psychiatrists	31	36	6	7

Test the claim that the distribution among psychiatrists in Canada is the same as the distribution published in the American study. Use a 5% significance level and the rejection region approach.

ANSWERS:

1. 30
2. (a) 840 (b) 720
3. 0.3
4. (a) 0.6467 (b) 0.0333 (c) 0.2577
(d) not independent since $P(\text{visual}) \neq P(\text{visual} \mid \text{good at math})$
5. 0.1346
6. (a) 0.2496 (b) 0.1469
7. (a) 0.97 (b) 1.7689
8. (a) 0.2033 (b) 0.4850 (c) 0.9988
9. (a) 0.8340 (b) 0.2088
10. (0.1154, 0.4046)
We are 85% confident that the average procrastination time online for all students is between 0.1154 and 0.4046 hours.
11. Sample should consist of at least 49 psychiatrists.
12. (a) $H_0 : p = 0.20$, $H_a : p \neq 0.20$
test statistic: $z^* = 1.3479$
 P -value ≈ 0.177
fail to reject H_0
(b) $n\hat{p} = 15 > 5$, $n\hat{q} = 40 > 5$
(c) There is insufficient evidence to claim that the proportion of board certified psychiatrists is different from 0.20, at the 10% significance level.
13. (a) $H_0 : \mu_1 = \mu_2$, $H_a : \mu_1 < \mu_2$
rejection region: $t < -2.462$
test statistic: $t^* = -3.6795$
reject H_0
(b) P -value < 0.005
(c) There is sufficient evidence to claim that male psychiatrists earn significantly more than female psychiatrists, at the 1% significance level.
14. (0.0684, 0.4134)
15. (a) $H_0 : d = 0$, $H_a : d > 0$
rejection region: $t > 2.132$
test statistic: $t^* = 1.4737$
fail to reject H_0
(b) P -value is between 0.10 and 0.125
16. H_0 : distribution of hours among psychiatrists in Canada fits the distribution of the American study
 H_a : distribution of hours among psychiatrists in Canada is different from the distribution of the American study
rejection region: $\chi^2 > 7.81$ ($d.f. = 3$)
test statistic: $\chi^2 = 8.1042$
reject H_0