

Question 1: (31 pts) Evaluate each of the following integrals, without the use of integration tables.

a) $\int \frac{3\sqrt[4]{x} + 6\sqrt[3]{x^5} - 4x^2}{2\sqrt{x}} dx$ b) $\int \frac{\csc^2(x)}{\cot(x) + 1} dx$ c) $\int \frac{(1 + \sqrt{x})^5}{\sqrt{x}} dx$ d) $\int_{-2}^0 \frac{x + 2}{\sqrt{x^2 + 4x + 9}} dx$
e) $\int \frac{x^4 + 5x^3 + 7x^2 + 2x + 1}{x^2 + 5x + 6} dx$ f) $\int_{1/3}^1 (1 - 6x) \ln(x) dx$ g) $\int (2x^2 + 1) 3^{4x^3 + 6x} dx$
h) $\int (2x^2 - x) \cos(3x) dx$

Question 2: (5 pts) Given the demand function $p_1(x) = 200 - 0.02x^2$ and the supply function $p_2(x) = x + 100$:

- Find the equilibrium point.
- Sketch and identify the regions representing the consumer and producer surpluses.
- Evaluate the producer surplus.

Question 3: (4 pts) Given the functions $f(x) = x^2 - 2x - 3$ and $g(x) = 13 - 2x$:

- Determine the point(s) of intersection of $f(x)$ and $g(x)$.
- Find the area of the region between $f(x)$ and $g(x)$, from $x = 0$ to $x = 5$.

Question 4: (4 pts) Use Simpson's rule to approximate $\int_0^3 \frac{1}{\sqrt{x^3 + 1}} dx$, using $n = 6$. Round your answer to 4 decimals.

Question 5: (9 pts) Use the table of integrals to solve the following. In each case, state the formula number and justify its use.

a) $\int_0^4 \frac{6}{1 + e^{0.5x}} dx$ b) $\int \frac{\sqrt{8 + 2x - x^2}}{x - 1} dx$ c) $\int \frac{4x}{x^4 - 4} dx$

Question 6: (3 pts) Find the demand function $p(x)$, given that the marginal revenue is given by $\frac{dR}{dx} = 9x^2 + 0.1x + 500$, and that the revenue for 10 units is \$8500.

Question 7: (4 pts) Solve the differential equation $xy' = (x^2 + 3)\sqrt{y}$, with the condition that $y = \frac{1}{9}$ when $x = 1$.

Question 8: (5 pts) A computer virus has already infected 4 million computers before an anti-virus software update is made available to remove it. After the update, the number of infected computers decreases at a rate that is proportional to the cube of the number of infected computers. One week after the update, there are still 2 million infected computers. Let N be the number of infected computers (in millions) and t be the number of weeks since the anti-virus update.

- a) Find the function $N(t)$ for the number of infected computers after t weeks.
b) When will there be 1 million computers infected?

Question 9: (6 pts) Evaluate the following limits:

a) $\lim_{x \rightarrow 1} \frac{x - e^{x-1}}{(x-1)^2}$ b) $\lim_{x \rightarrow 0} \frac{x^2 - 3x + 3 \sin(x)}{\cos(x) - 1}$

Question 10: (8 pts) Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

a) $\int_0^1 \frac{e^{-\frac{1}{x}}}{x^2} dx$ b) $\int_0^{+\infty} \frac{2x}{x^2 + 3} dx$

Question 11: (3 pts) Consider the sequence $\left\{ \frac{3}{-4}, \frac{6}{8}, \frac{9}{-16}, \frac{12}{32}, \dots \right\}$

- a) Give the 5th term of the sequence. b) Find an expression for the n^{th} term of the sequence.

Question 12: (6 pts) Determine if the following sequences converge or diverge. If the sequence converges, find the limit.

a) $a_n = \frac{2(2n+1)!}{(2n+3)!}$ b) $a_n = \frac{2^n}{4n+7}$

Question 13: (6 pts) Determine if the following series converge or diverge. If the series converges, find its sum.

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{n+3}$ b) $\sum_{n=0}^{\infty} \frac{3(2^n) + 4}{4^n}$

Question 14: (3 pts) A deposit of \$10 is made every week for a period of 12 years, in an account that earns 2% interest per year, compounded weekly. Find the balance in the account after 12 years.

Question 15: (3 pts) Given the number $8.2\overline{41}$, express it using a geometric series. Find the sum of the geometric series to write the number as the ratio of two integers.

ANSWERS

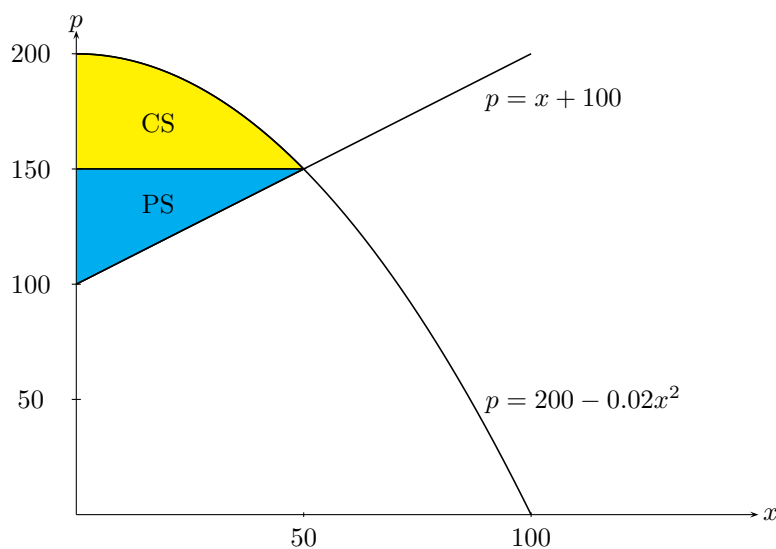
1.) a) $2x^{3/4} + \frac{18}{13}x^{13/6} - \frac{4}{5}x^{5/2} + C$ b) $-\ln|\cot(x) + 1| + C$ c) $\frac{1}{3}(1 + \sqrt{x})^6 + C$ d) $3 - \sqrt{5}$

e) $\frac{1}{3}x^3 + x + \ln|x+2| - 4\ln|x+3| + C$ f) $\frac{2}{3}$ g) $\frac{1}{6\ln(3)} 3^{4x^3+6x} + C$

h) $\frac{2x^2 - x}{3} \sin(3x) + \frac{4x - 1}{9} \cos(3x) - \frac{4}{27} \sin(3x) + C$

2.) a) $E=(50, 150)$

b)



c) $PS = \$1250$

3.) a) $(4,5)$ and $(-4,21)$ b) 47 units^2 4.) 1.6557

5.) a) $24 - 12\ln(1 + e^2) + 12\ln(2)$ b) $\sqrt{8 + 2x - x^2} - 3\ln\left|\frac{3 + \sqrt{8 + 2x - x^2}}{x - 1}\right| + C$

c) $\frac{1}{2}\ln\left|\frac{x^2 - 2}{x^2 + 2}\right| + C$ 6.) $p(x) = 3x^2 + 0.05x + 500 + \frac{495}{x}$ 7.) $y = \left(\frac{x^2}{4} + \frac{3}{2}\ln|x| + \frac{1}{12}\right)^2$

8.) a) $N(t) = \frac{4}{\sqrt{3t+1}}$ b) 5 weeks 9.) a) $\frac{-1}{2}$ b) -2

10.) a) converges to $\frac{1}{e}$ b) diverges 11.) a) $a_5 = \frac{-15}{64}$ b) $a_n = (-1)^n \frac{3n}{2^{n+1}}$

12.) a) converges to 0 b) diverges 13.) diverges b) converges to $\frac{34}{3}$

14.) $\$7053.66$ 15.) $8.2 + \sum_{n=0}^{+\infty} \frac{41}{1000} \left(\frac{1}{100}\right)^n = \frac{8159}{990}$