Question 1: (*31 pts*) Evaluate each of the following integrals, without the use of integration tables.

a) $\displaystyle\int \frac{3\sqrt[4]{x}+6\sqrt[3]{x^5}-4x^2}{2\sqrt{x}}\,dx$ **b)** $\displaystyle\int \frac{\csc^2(x)}{\cot(x)+1}\,dx$ **c)** $\displaystyle\int \frac{(1+\sqrt{x})^5}{\sqrt{x}}\,dx$ **d)** $\displaystyle\int_{-2}^{0} \frac{x+2}{\sqrt{x^2+4x+9}}\,dx$

e) $\displaystyle\int \frac{x^4+5x^3+7x^2+2x+1}{x^2+5x+6}\,dx$ **f)** $\displaystyle\int_{1/3}^{1} (1-6x)\ln(x)\,dx$ **g)** $\displaystyle\int \left(2x^2+1\right)\,3^{4x^3+6x}\,dx$

h) $\displaystyle\int \left(2x^2-x\right)\,\cos(3x)\,dx$

Question 2: (*5 pts*) Given the demand function $p_1(x) = 200 - 0.02x^2$ and the supply function $p_2(x) = x + 100$:

a) Find the equilibrium point.

b) Sketch and identify the regions representing the consumer and producer surpluses.

c) Evaluate the producer surplus.

Question 3: (*4 pts*) Given the functions $f(x) = x^2 - 2x - 3$ and $g(x) = 13 - 2x$:

a) Determine the point(s) of intersection of $f(x)$ and $g(x)$.

b) Find the area of the region between $f(x)$ and $g(x)$, from $x = 0$ to $x = 5$.

Question 4: (*4 pts*) Use Simpson's rule to approximate $\displaystyle\int_0^3 \frac{1}{\sqrt{x^3+1}}\,dx$, using $n = 6$. Round your answer to 4 decimals.

Question 5: (*9 pts*) Use the table of integrals to solve the following. In each case, state the formula number and justify its use.

a) $\displaystyle\int_0^4 \frac{6}{1+e^{0.5x}}\,dx$ **b)** $\displaystyle\int \frac{\sqrt{8+2x-x^2}}{x-1}\,dx$ **c)** $\displaystyle\int \frac{4x}{x^4-4}\,dx$

Question 6: (*3 pts*) Find the demand function $p(x)$, given that the marginal revenue is given by $\dfrac{dR}{dx} = 9x^2 + 0.1x + 500$, and that the revenue for 10 units is \$8500.

Question 7: (*4 pts*) Solve the differential equation $xy' = (x^2+3)\sqrt{y}$, with the condition that $y = \dfrac{1}{9}$ when $x = 1$.

Question 8: (*5 pts*) A computer virus has already infected 4 million computers before an anti-virus software update is made available to remove it. After the update, the number of infected computers decreases at a rate that is proportional to the cube of the number of infected computers. One week after the update, there are still 2 million infected computers. Let N be the number of infected computers (in millions) and t be the number of weeks since the anti-virus update.

a) Find the function $N(t)$ for the number of infected computers after t weeks.

b) When will there be 1 million computers infected?

Question 9: (*6 pts*) Evaluate the following limits:

a) $\lim\limits_{x\to 1} \dfrac{x - e^{x-1}}{(x-1)^2}$ **b)** $\lim\limits_{x\to 0} \dfrac{x^2 - 3x + 3\sin(x)}{\cos(x) - 1}$

Question 10: (*8 pts*) Determine whether the following improper integrals converge or diverge. If the integral converges, find its value.

a) $\displaystyle\int_0^1 \frac{e^{\frac{-1}{x}}}{x^2}\,dx$ **b)** $\displaystyle\int_0^{+\infty} \frac{2x}{x^2+3}\,dx$

Question 11: (*3 pts*) Consider the sequence $\left\{\dfrac{3}{-4}, \dfrac{6}{8}, \dfrac{9}{-16}, \dfrac{12}{32}, \cdots\right\}$

a) Give the 5^{th} term of the sequence. **b)** Find an expression for the n^{th} term of the sequence.

Question 12: (*6 pts*) Determine if the following sequences converge or diverge. If the sequence converges, find the limit.

a) $a_n = \dfrac{2\,(2n+1)!}{(2n+3)!}$ **b)** $a_n = \dfrac{2^n}{4n+7}$

Question 13: (*6 pts*) Determine if the following series converge or diverge. If the series converges, find its sum.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+4}}{n+3}$ **b)** $\displaystyle\sum_{n=0}^{\infty} \frac{3\,(2^n)+4}{4^n}$

Question 14: (*3 pts*) A deposit of \$10 is made every week for a period of 12 years, in an account that earns 2% interest per year, compounded weekly. Find the balance in the account after 12 years.

Question 15: (*3 pts*) Given the number $8.2\overline{41}$, express it using a geometric series. Find the sum of the geometric series to write the number as the ratio of two integers.

ANSWERS

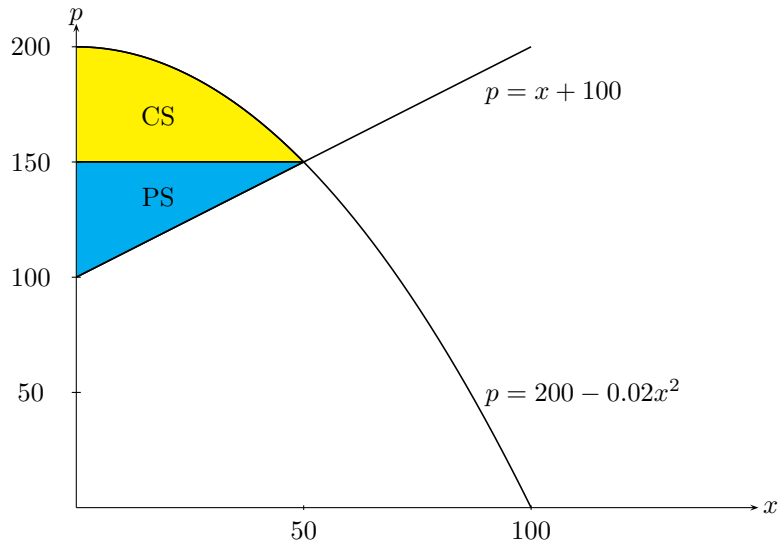
1.) a) $2x^{3/4} + \frac{18}{13}x^{13/6} - \frac{4}{5}x^{5/2} + C$ b) $-\ln|\cot(x) + 1| + C$ c) $\frac{1}{3}(1+\sqrt{x})^6 + C$ d) $3 - \sqrt{5}$

e) $\frac{1}{3}x^3 + x + \ln|x+2| - 4\ln|x+3| + C$ f) $\frac{2}{3}$ g) $\frac{1}{6\ln(3)}\, 3^{4x^3+6x} + C$

h) $\frac{2x^2 - x}{3}\sin(3x) + \frac{4x-1}{9}\cos(3x) - \frac{4}{27}\sin(3x) + C$

2.) a) E=$(50, 150)$

b)



c) PS = \$1250

3.) a) (4,5) and (-4,21) b) 47 units^2 **4.)** 1.6557

5.) a) $24 - 12\ln(1+e^2) + 12\ln(2)$ b) $\sqrt{8+2x-x^2} - 3\ln\left|\frac{3+\sqrt{8+2x-x^2}}{x-1}\right| + C$

c) $\frac{1}{2}\ln\left|\frac{x^2-2}{x^2+2}\right| + C$ **6.)** $p(x) = 3x^2 + 0.05x + 500 + \frac{495}{x}$ **7.)** $y = \left(\frac{x^2}{4} + \frac{3}{2}\ln|x| + \frac{1}{12}\right)^2$

8.) a) $N(t) = \frac{4}{\sqrt{3t+1}}$ b) 5 weeks **9.)** a) $\frac{-1}{2}$ b) -2

10.) a) converges to $\frac{1}{e}$ b) diverges **11.)** a) $a_5 = \frac{-15}{64}$ b) $a_n = (-1)^n \frac{3n}{2^{n+1}}$

12.) a) converges to 0 b) diverges **13.)** diverges b) converges to $\frac{34}{3}$

14.) \$7053.66 **15.)** $8.2 + \sum_{n=0}^{+\infty} \frac{41}{1000}\left(\frac{1}{100}\right)^n = \frac{8159}{990}$