Marks 1. Simplify each expression completely. Your final answer must contain positive exponents only.

[1.5] (a) $\left(\dfrac{a^3b^2}{2a^{-3}b^5}\right)^2 \dfrac{a^4b}{\sqrt[3]{8}}$

$= \dfrac{a^6\ b^4}{4\ a^{-6}\ b^{10}} \cdot \dfrac{a^4\ b}{2}$

$= \frac{1}{8}\ a^{16}\ b^{-5}$

$= \dfrac{a^{16}}{8\ b^5}$

[1.5] (b) $\sqrt[3]{24\,x^4y^6z^5}$

$= 2\,x\,y^2\,z\ \sqrt[3]{3\,x\,z^2}$

Marks 2. Find the exact value without using a calculator:

[1.5] (a) $\left(\sqrt{12}-5\sqrt{3}\right)^2$

$= (\sqrt{12})^2 - 2\sqrt{12}\cdot 5\sqrt{3} + (5\sqrt{3})^2$

$= 12 - 10\sqrt{36} + 25(3)$

$= 12 - 60 + 75$

$= 27$

[1.5] (b) $(-8)^{\frac{1}{3}} + 2(4)^{\frac{3}{2}} - 6(1)^{\frac{1}{3}}$

$= -2 + 2(8) - 6$

$= -2 + 16 - 6$

$= 8$

[2] 3. Rationalize the numerator and simplify: $\dfrac{3-\sqrt{25-x^2}}{x+4}$

$$= \frac{(3-\sqrt{25-x^2})(3+\sqrt{25-x^2})}{(x+4)(3+\sqrt{25-x^2})}$$

$$= \frac{9-(25-x^2)}{(x+4)(3+\sqrt{25-x^2})}$$

$$= \frac{x^2-16}{(x+4)(3+\sqrt{25-x^2})}$$

$$= \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(3+\sqrt{25-x^2})}$$

$$= \frac{x-4}{3+\sqrt{25-x^2}}$$

Marks 4. Factor each polynomial compeletely:

[2] (a) $x^3 + x^2y - 9x - 9y$

$$= x^2(x+y) - 9(x+y)$$
$$= (x+y)(x^2 - 9)$$
$$= (x+y)(x+3)(x-3)$$

[2] (b) $x^4 - 16$

$$= (x^2)^2 - 4^2$$
$$= (x^2+4)(x^2-4)$$
$$= (x^2+4)(x+2)(x-2)$$

[2] (c) $27x^3 + 8$

$$= (3x)^3 + 2^3$$
$$= (3x+2)\left((3x)^2 - (3x)(2) + 2^2\right)$$
$$= (3x+2)(9x^2 - 6x + 4)$$

Marks 5. Perform the indicated operations and simplify the results:

[2] (a) $(2x-5y)(4x^2+10xy+25y^2)$

$= 8x^3 + \cancel{20x^2y} + \cancel{50xy^2} - \cancel{20x^2y} - \cancel{50xy^2} - 125y^3$

$= 8x^3 - 125y^3$

[3] (b) $\dfrac{5x^2+25x+30}{x^2+6x+9} \div \dfrac{x^2-4}{x+3}$

$= \dfrac{5(x^2+5x+6)}{x^2+6x+9} \cdot \dfrac{x+3}{x^2-4}$

$= \dfrac{5\cancel{(x+2)}\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+3)}} \cdot \dfrac{\cancel{(x+3)}}{\cancel{(x+2)}(x-2)}$

$= \dfrac{5}{x-2}$

[3] (c) $\dfrac{x+1}{x^2+5x} - \dfrac{2x-1}{2x^2+9x-5}$

$= \dfrac{x+1}{x(x+5)} - \dfrac{2x-1}{(x+5)(2x-1)}$

$= \dfrac{(x+1)(2x-1) - (2x-1)x}{x(x+5)(2x-1)}$

$= \dfrac{2x^2+x-1-2x^2+x}{x(x+5)(2x-1)}$

$= \dfrac{\cancel{2x-1}}{x(x+5)\cancel{(2x-1)}}$

$= \dfrac{1}{x(x+5)}$

Marks

[2] 5. (d) $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{y}-\dfrac{1}{x}}$

$$= \frac{x^2-y^2}{xy} \div \frac{x-y}{xy}$$

$$= \frac{(x+y)\cancel{(x-y)}}{\cancel{xy}} \cdot \frac{\cancel{xy}}{\cancel{x-y}}$$

$$= x+y$$

[2] 6. Use long division to find the quotient and the remainder of

$$\frac{x^4+x^3-8x^2+5x+1}{x^2+2x}$$

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x^2+2x & \cancel{x^4} + x^3 - 8x^2 + 5x + 1 \\ & \cancel{x^4} + 2x^3 \\ \hline & -\cancel{x^3} - 8x^2 \\ & -\cancel{x^3} - 2x^2 \\ \hline & -\cancel{6x^2} + 5x \\ & -\cancel{6x^2} - 12x \\ \hline & 17x + 1 \end{array}$$

$$\therefore \frac{x^4+x^3-8x^2+5x+1}{x^2+2x} = x^2 - x - 6 + \frac{17x+1}{x^2+2x}$$

Marks

7. Given the points P(-1,2) and Q(2,3), find the exact value of the following:

[1] (a) the distance d between points P and Q

$$d = \sqrt{(2+1)^2 + (3-2)^2}$$
$$= \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$

[1] (b) the midpoint of the line segment joining P and Q

$$M = \left(\frac{-1+2}{2}, \frac{2+3}{2}\right)$$
$$= \left(\frac{1}{2}, \frac{5}{2}\right)$$

[1.5] (c) the equation of the straight line L_1 passing through P and Q

$$m = \frac{3-2}{2+1} = \frac{1}{3}$$
$$y = mx + b \Rightarrow 2 = \frac{1}{3}(-1) + b$$
$$\Rightarrow b = 2 + \frac{1}{3} = \frac{7}{3}$$

$\therefore$ Equation of L_1: $y = \frac{1}{3}x + \frac{7}{3}$

OR $x - 3y + 7 = 0$

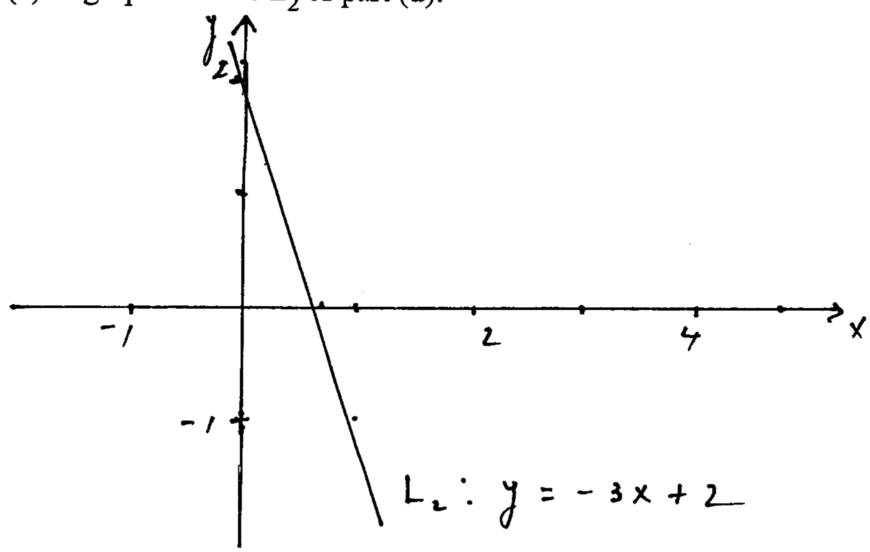
[1] (d) the equation of the line L_2 that is perpendicular to L_1 and has y-intercept 2

The slope of line L_2 is -3

$\therefore$ Equation of line L_2: $y = -3x + 2$

OR $3x + y - 2 = 0$

[1.5] 7. (e) graph the line L_2 of part (d).



8. Solve for x:

[1.5] (a) $15 - x(6-2x) = 2x\,(x+2)$

$15 - 6x + 2x^2 = 2x^2 + 4x$

$-10x = -15$

$x = \frac{3}{2}$

[2] (b) $2x^2 = x + 6$

$2x^2 - x - 6 = 0$

$(x-2)(2x+3) = 0$

$x = 2$ or $x = -\frac{3}{2}$

[2] (c) $x^2 - 2x = 10$

$x^2 - 2x - 10 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-10)}}{2(1)}$

$= \frac{2 \pm \sqrt{4 + 40}}{2}$

$= \frac{2 \pm \sqrt{44}}{2} = \frac{2 \pm 2\sqrt{11}}{2}$

$\therefore x = 1 - \sqrt{11}$ or $x = 1 + \sqrt{11}$

Marks

[2.5] 8. (d) $\frac{x}{x^2-1} - \frac{1}{x^2-x} = \frac{1}{x^2+x}$

$$\frac{x}{(x+1)(x-1)} - \frac{1}{x(x-1)} = \frac{1}{x(x+1)}$$

$$\frac{x \cdot x - (x+1) - (x-1)}{x(x-1)(x+1)} = 0$$

$$x^2 - x - 1 - x + 1 = 0$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0 \quad \text{or} \quad x = 2$$

$x = 0$ does not check

solution: $x = 2$

[1.5] (e) $\sqrt{4x+5} = x$

$$\left(\sqrt{4x+5}\right)^2 = x^2$$

$$4x + 5 = x^2$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = 5 \quad \text{or} \quad x = -1$$

$x = -1$ does not check

Solution: $x = 5$

[1.5] 8. (f) $5(3-2x) \geq 2x+21$

$$15-10x \geq 2x+21$$
$$-12x \geq 6$$
$$x \leq -\frac{6}{12}$$
$$x \leq -\frac{1}{2}$$

[2] (g) $(10)^{4x} = (100)^{x-1}$

$$(10)^{4x} = (10^2)^{x-1}$$
$$(10)^{4x} = (10)^{2(x-1)}$$
$$\therefore \quad 4x = 2(x-1)$$
$$4x = 2x-2$$
$$2x = -2$$
$$x = -1$$

Solution : $x = -1$

Marks

[1] 8. (h) $e^{2x}=3$

$$2x = \ln 3$$
$$2x = 1.0986$$
$$x = \frac{1.0986}{2}$$
$$= 0.5493$$

[2] (i) $\log_2 x + \log_2(x+5) = \log_2 6$

$$\log_2 x(x+5) = \log_2 6$$
$$x(x+5) = 6$$
$$x^2 + 5x - 6 = 0$$
$$(x+6)(x-1) = 0$$
$$x = -6 \text{ or } x = 1$$

$x = -6$ does not check

Solution : $x = 1$

Marks

[2] 9. Given $f(x) = x^2 + x$, find and **simplify** $\frac{f(x+h)-f(x)}{h}$

$$= \frac{\left[(x+h)^2 + (x+h)\right] - \left[x^2 + x\right]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 \cancel{+x} + h \cancel{-x^2} \cancel{-x}}{h}$$

$$= \frac{2xh + h^2 + h}{h}$$

$$= 2x + h + 1$$

[1] 10. Find the **domain** and **range** of the function $f(x) = \sqrt{1-x}$.

$1 - x \geq 0 \implies x \leq 1$

$\therefore$ Domain $= (-\infty, 1]$

Range $= [0, \infty)$

Marks

11. Given the function $f(x) = x^2 - 4x + 3$,

[1] (a) use **completing the square method** to write it in the form $f(x) = a(x-h)^2 + k$ and then state the following:

$$x^2 - 4x + 3 = x^2 - 4x + 4 - 4 + 3$$
$$= (x-2)^2 - 1$$

$\therefore f(x) = (x-2)^2 - 1$ $a = 1,\ h = 2,\ k = -1$

[0.5] (b) the vertex : $(2, -1)$

[1.5] (c) the x- and y-intercept(s)

$y = 0 \Rightarrow x^2 - 4x + 3 = 0$
$(x-1)(x-3) = 0$

$\therefore$ x-intercepts : $(1, 0)$, $(3, 0)$

y-intercept : $(0, 3)$

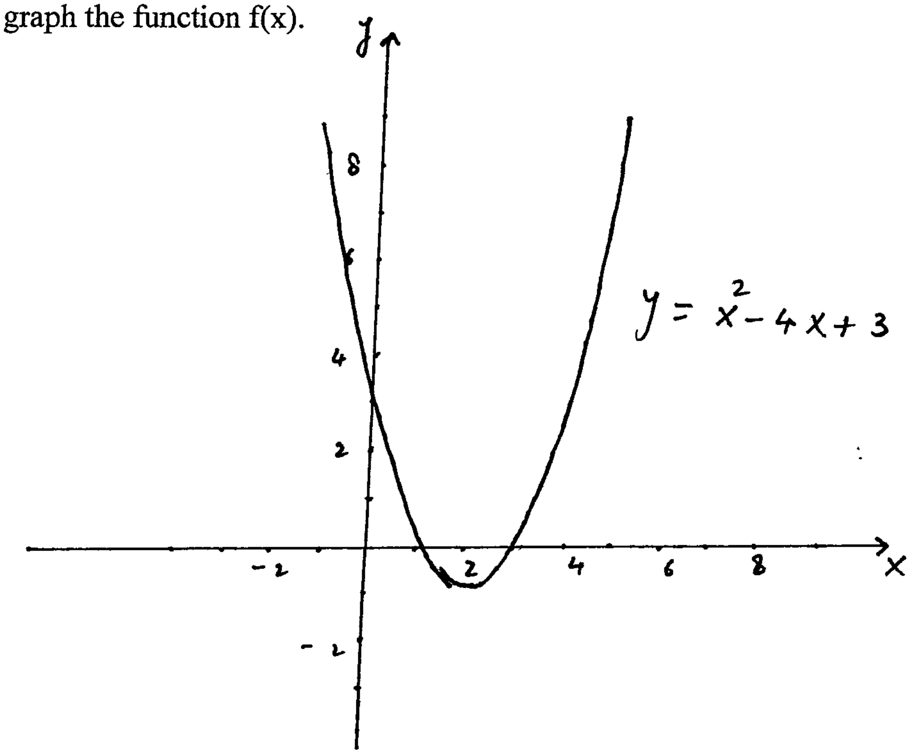
[0.5] (d) the equation of the axis of symmetry

$x = 2$

[0.5] (e) the range

$[-1, \infty)$

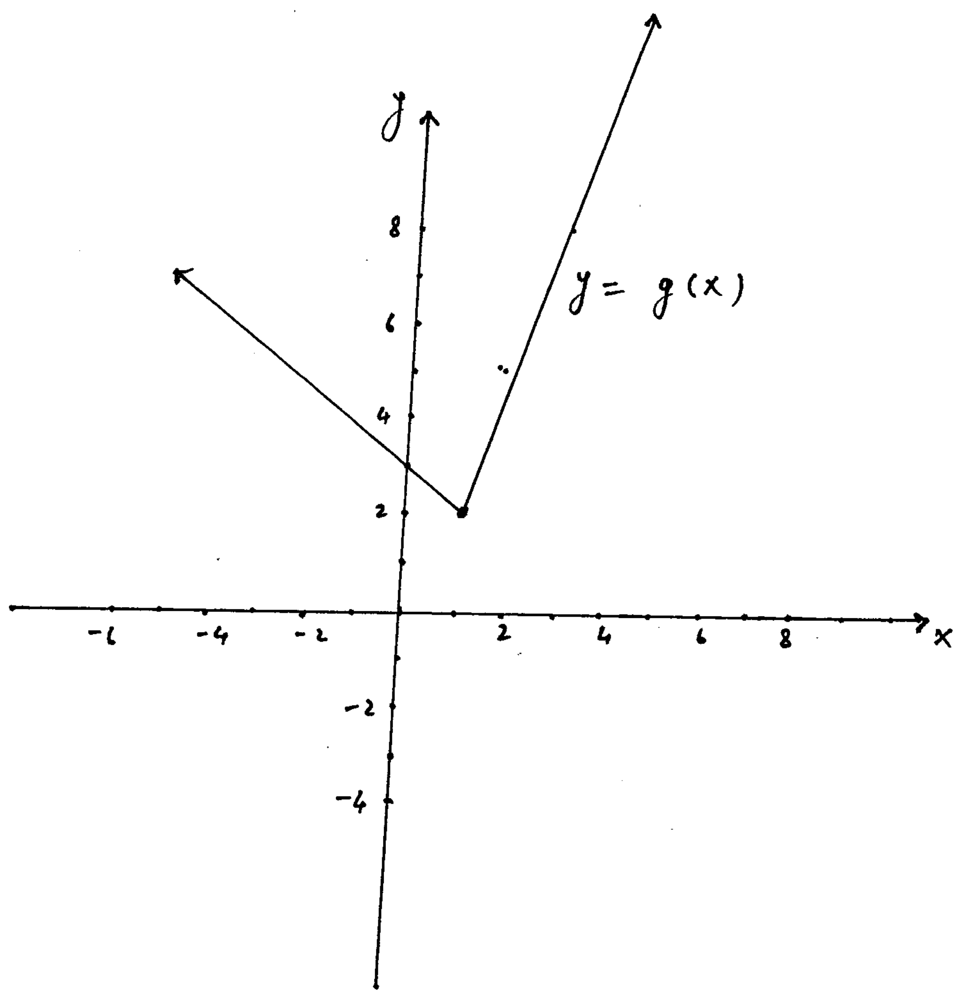
[2] (f) graph the function f(x).



Marks

[2] 12. Graph the function,

$$g(x) = \begin{cases} -x+3, & x<1 \\ 3x-1, & x \geq 1 \end{cases}$$



Marks

13. Given $f(x) = \frac{4}{x}$ and $g(x) = \frac{2}{3x-5}$, find the following:

[1] (a) $(f/g)(x) = \frac{f(x)}{g(x)}$

$$= \frac{4/x}{\frac{2}{3x-5}} = \frac{4}{x} \cdot \frac{3x-5}{2}$$

$$= \frac{2(3x-5)}{x}$$

[1] (b) $f \circ g = f(g(x))$

$$= f\left(\frac{2}{3x-5}\right)$$

$$= \frac{4}{\frac{2}{3x-5}}$$

$$= \frac{4 \cdot (3x-5)}{2}$$

$$= 2(3x-5)$$

[1] (c) $g^{-1}(x)$

$$y = \frac{2}{3x-5}$$

inverse : $x = \frac{2}{3y-5}$

$$3xy - 5x = 2$$

$$3xy = 2 + 5x$$

$$y = \frac{2+5x}{3x}$$

$$\therefore \quad g^{-1}(x) = \frac{2+5x}{3x}$$

Marks

[4] 14. Given the function $y = \dfrac{2x-4}{x-1}$,

(a) state the domain and range, find the equations of the vertical and horizontal asymptotes and the x and y-intercepts

Domain $= (-\infty, 1) \cup (1, \infty)$

Range $= (-\infty, 2) \cup (2, \infty)$

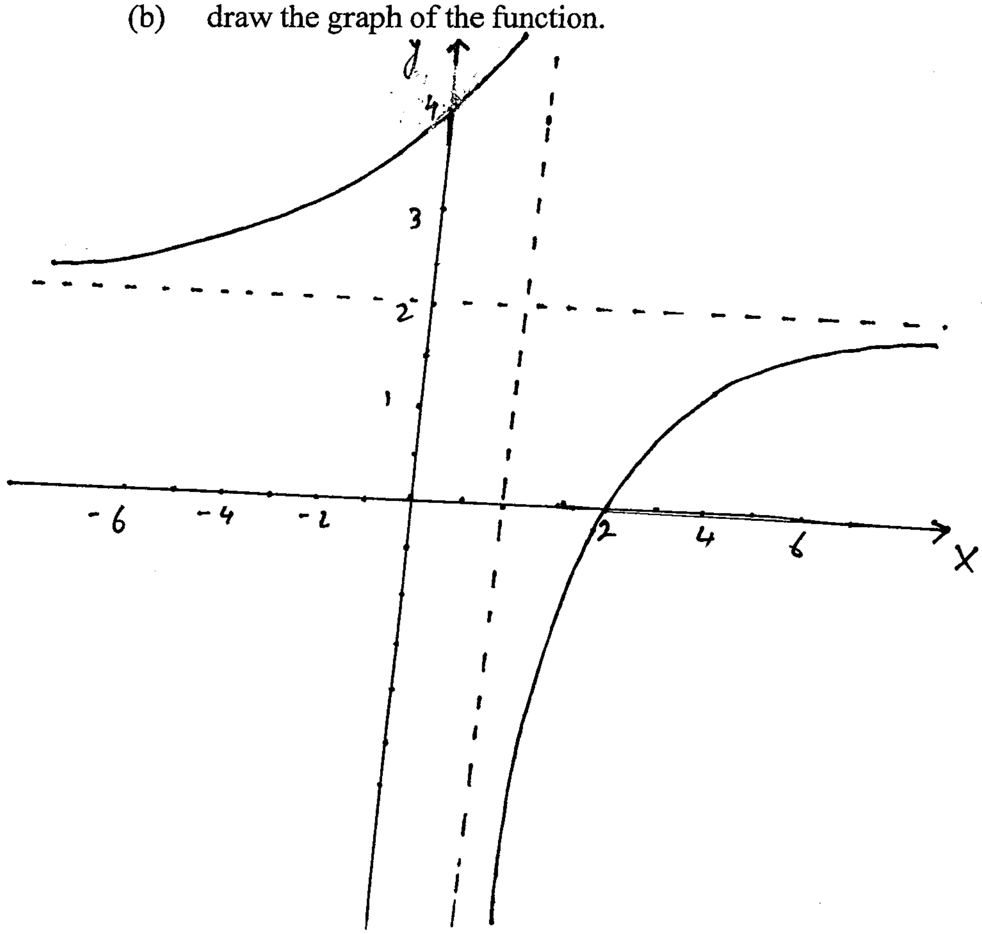
V.A. : $x = 1$

H.A : $y = 2$

x-int : $(2, 0)$

y-int. : $(0, 4)$

(b) draw the graph of the function.



Marks

15. Given $y = -1 + 3^x$

[0.5] (a) find the equation of the asymptote

H.A : $y = -1$

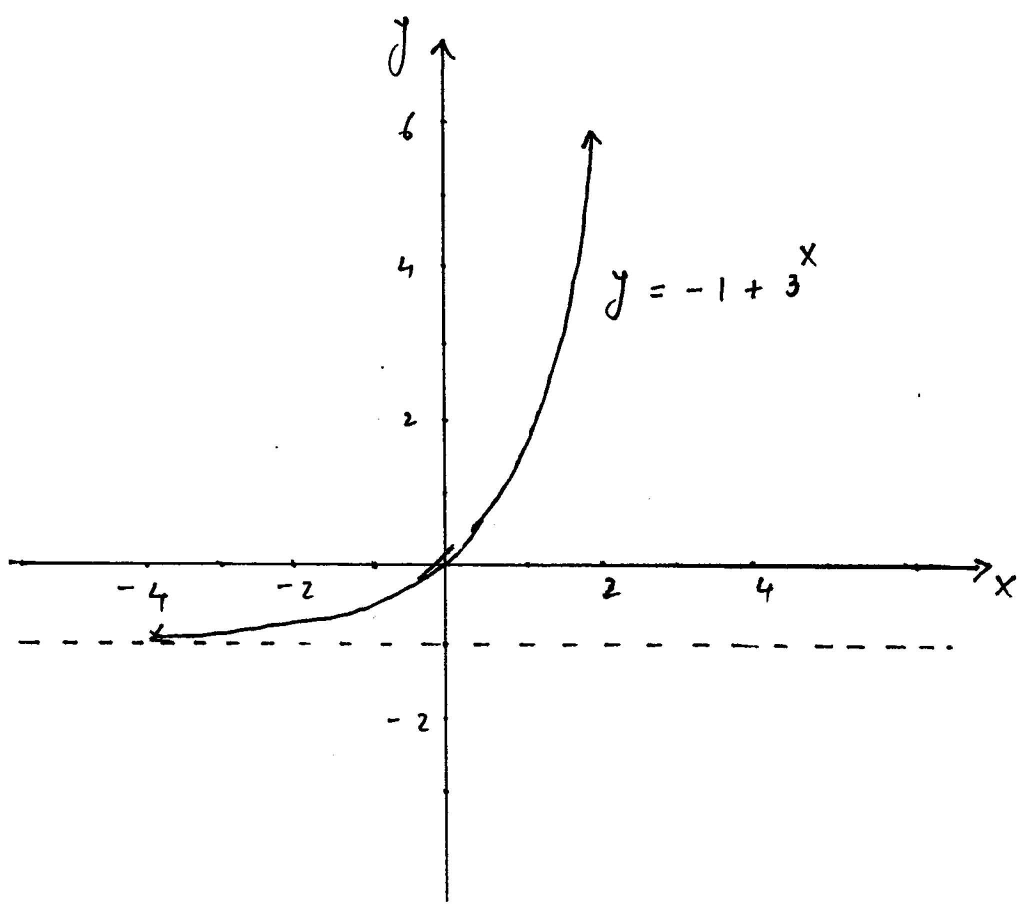
[1] (b) the x and y-intercepts

x-int. : $(0, 0)$

y-int. : $(0, 0)$

[0.5] (c) state the range : $(-1, \infty)$

[2] (d) sketch the graph of the function.



Marks

[2] 16. If $2000 is invested at 3% annual rate of interest, compounded monthly, what will the value of the invest be in 5 years? Round your answer to the nearest cent.

$P = 2000, \quad r = 0.03, \quad n = 12, \quad t = 5$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 2000\left(1 + \frac{0.03}{12}\right)^{12(5)}$

$= 2000(1.0025)^{60}$

$= \$2323.23$

[2] 17. Find the value of the following to four decimal places:

(a) $\ln 9$ $= 2.1972$

(b) $\log_6(12)$ $= \frac{\ln 12}{\ln 6} = \frac{2.4849}{1.7918}$

$= 1.3868$

[1] 18. Rewrite as a single logarithm: $3\ln(5) + \frac{1}{2}\ln y - 2\ln x$

$= \ln\left(\frac{5^3 y^{1/2}}{x^2}\right)$

Marks

[1] 19. Rewrite as sum/difference of multiples of logarithms: $\log_5 \dfrac{x(x+2)^4}{\sqrt[3]{x+3}}$

$= \log_5 x + 4\log_5 (x+2) - \frac{1}{3}\log_5 (x+3)$

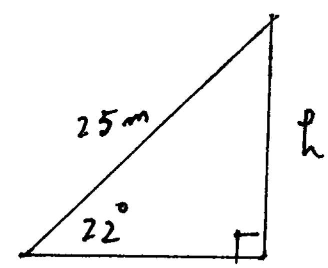
[2] 20. A kite is caught in the top branches of a tree. If the 25 meter kite string makes an angle of 22^0 with the ground, estimate the height of the tree by finding the distance from the kite to the ground.

$\frac{h}{25} = \sin 22^\circ$

$h = 25 \sin 22^\circ$

$= 25\,(0.3746)$

$= 9.365\ m$



[1] 21. Find the **complement** and **supplement** of 75^0 angle.

Complement $= 90^\circ - 75^\circ = 15^\circ$

Supplement $= 180^\circ - 75^\circ = 105^\circ$

Marks

22. Convert:

[1] (a) 36^o to radian measure. Leave your answer in terms of π

$$36^\circ = 36^\circ \cdot \frac{\pi}{180^\circ}$$

$$= \frac{\pi}{5}$$

[1] (b) $-3\pi/5$ radians to degree measure

$$\frac{-3\pi}{5} = \frac{-3\pi}{5} \cdot \frac{180^\circ}{\pi}$$

$$= -108^\circ$$

[2] 23. Draw a picture, state the reference angle and give the exact value for

(a) $\sec(5\pi/3)$ $= \sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos(\pi/3)} = \frac{1}{1/2} = 2$

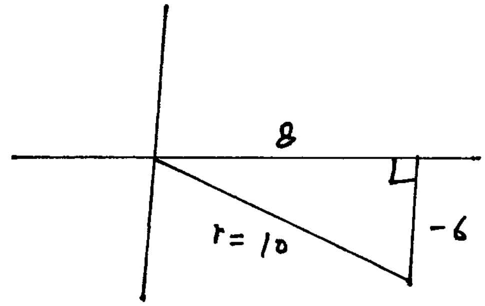
(b) $\cot(-210^o)$ $= -\cot 30^\circ = -\sqrt{3}$

[2] 24. If θ is an angle in standard position whose terminal side contains the point (8,-6), find

(a) csc θ

$= -\frac{10}{6}$

$= -\frac{5}{3}$



(b) cos θ

$r = \sqrt{8^2 + (-6)^2} = 10$

$= \frac{8}{10} = \frac{4}{5}$

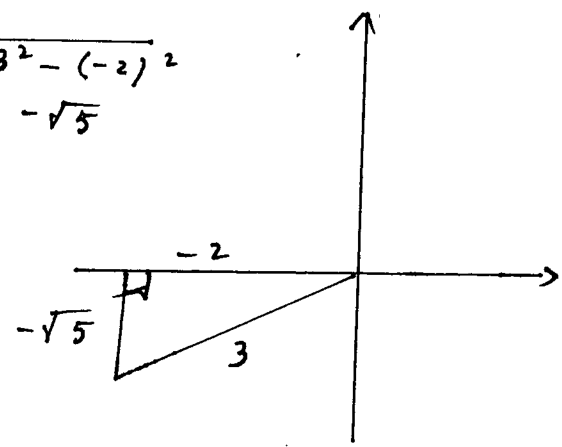
[2] 25. Given that cos θ = -2/3 and sin θ < 0, find the exact value of

(a) tan θ

$y = -\sqrt{3^2 - (-2)^2}$
$= -\sqrt{5}$

$= \frac{-\sqrt{5}}{-2}$

$= \frac{\sqrt{5}}{2}$



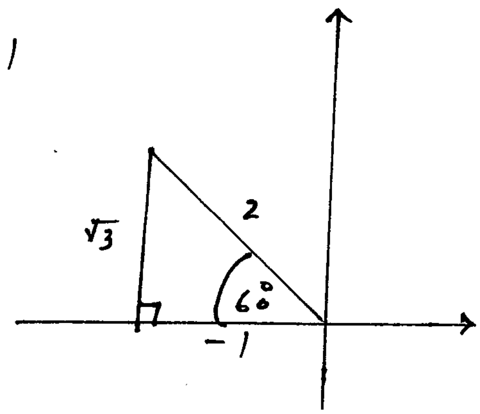
(b) sin θ

$= -\frac{\sqrt{5}}{3}$

[1] 26. Find the exact value of the angle θ in $[0^{\circ}, 360^{0})$ for which $\sin\theta = \sqrt{3}/2$ and θ is in QII.

$x = -\sqrt{2^2 - (\sqrt{3})^2} = -1$

$\therefore \theta = 120^{\circ}$



[3] 27. Verify the following identities:

(a) $\cos\theta\,(\sec\theta - \cos\theta) = \sin^2\theta$

$\cos\theta\,(\sec\theta - \cos\theta)$

$= \cos\theta\cdot\sec\theta - \cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta$

(b) $\dfrac{1}{\tan\theta + \cot\theta} = \sin\theta\ \cos\theta$

$\dfrac{1}{\tan\theta + \cot\theta}$

$= \dfrac{1}{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\cdot\sin\theta}} = \dfrac{\cos\theta\ \sin\theta}{\sin^2\theta + \cos^2\theta} = \dfrac{\cos\theta\ \sin\theta}{1}$

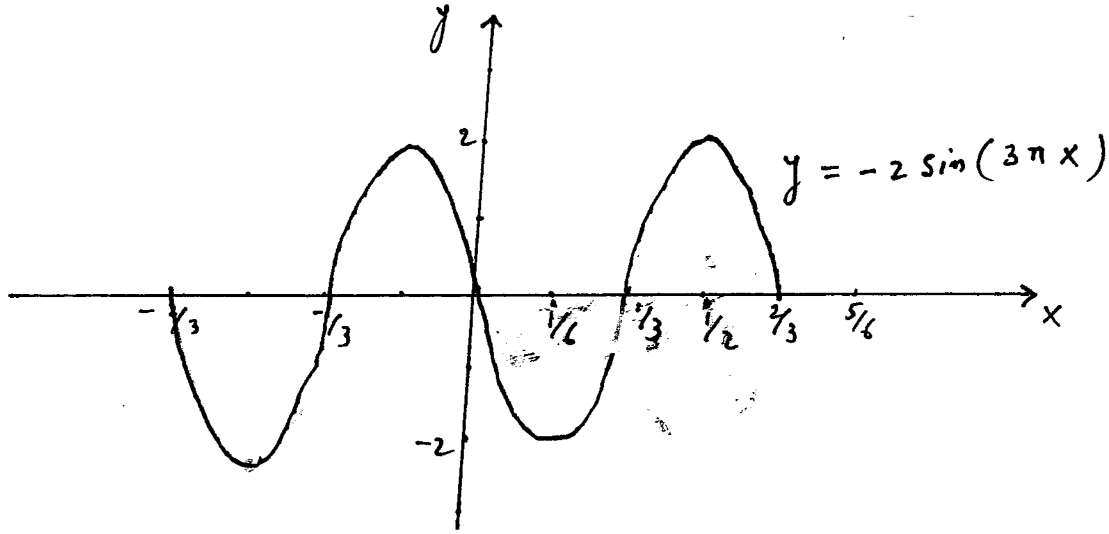
$= \cos\theta\ \sin\theta$

Marks

[3] 28. State the amplitude and period, then graph two cycles of $f(x) = -2\sin(3\pi x)$.

$$\text{Amplitude} = |-2| = 2, \quad \text{Period} = \frac{2\pi}{3\pi} = \frac{2}{3}$$

x	$-4/6$	$-3/6$	$-2/6$	$-1/6$	0	$1/6$	$2/6$	$3/6$	$4/6$
$3\pi x$	-2π	$-3\pi/2$	$-\pi$	$-\pi/2$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = -2\sin(3\pi x)$	0	-2	0	2	0	-2	0	2	0



[2] 29. In a triangle ABC, angle A=30° and angle C=78° while side a = 8. Find side c.

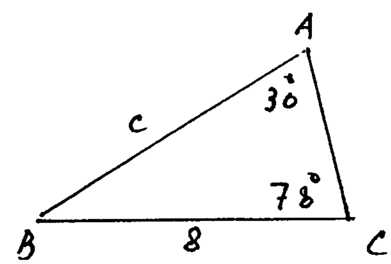
$$\frac{c}{\sin 78^\circ} = \frac{8}{\sin 30^\circ}$$

$$c = \frac{8 \sin 78^\circ}{\sin 30^\circ}$$

$$= \frac{8(0.9781)}{1/2}$$

$$= 16(0.9781)$$

$$= 15.65$$



Marks

[2] 30. In a triangle ABC, a=7, b = 9, and c = 6. Find angle C.

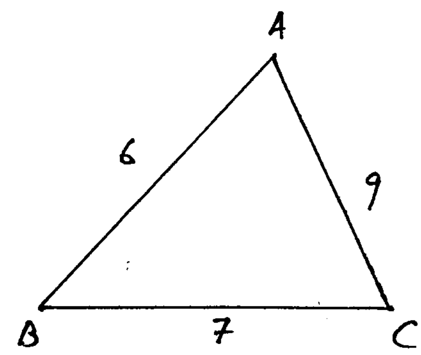
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{7^2 + 9^2 - 6^2}{2 \cdot 7 \cdot 9}$$

$$= 0.7460$$

$$\therefore C = \arccos(0.7460)$$

$$= 41.75^\circ$$



[2] 31. Two automobiles leave from the same point and travel along straight highways which differ in direction by 82°. If their speeds are 100 km/hour and 120 km/hour, how far apart will they be, to the nearest meter, an hour later?

$$x^2 = (120)^2 + (100)^2 - 2(120)(100)\cos 82^\circ$$

$$= 14400 + 10000 - 24000(0.1392)$$

$$= 21{,}059.2$$

$$x = \sqrt{21{,}059.2}$$

$$= 145.1179 \text{ km}$$

$$= 145118 \text{ m}$$