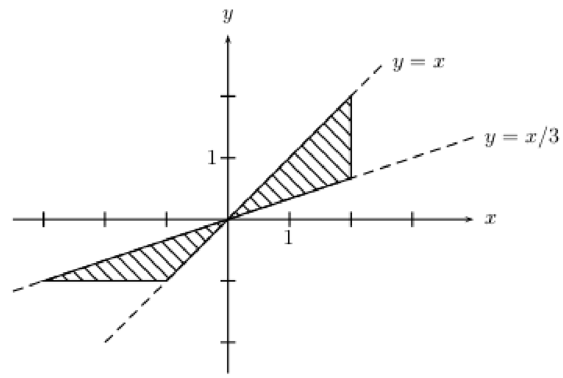
[Marks]

1. Given $f(x) = arc\sec\sqrt{x^2+1}$, assuming $x > 0$ [3]
   a. Find $\frac{dy}{dx}$
   b. Simplify your answer.
2. Evaluate the integrals. [30]
   a. $\int x\ \sqrt[3]{x-1}\ dx$
   b. $\int_{2/5}^{4/5} \frac{\sqrt{25x^2-4}}{x}\ dx$
   c. $\int t^2 \arcsin t\ dt$
   d. $\int (a+\tan x)^2\ dx$
   e. $\int \frac{dx}{\sqrt{2x-x^2}}$
   f. $\int \frac{x^2+2x+5}{x^2(x^2+1)}\ dx$
3. Evaluate the improper integrals. [12]
   a. $\int_{-\infty}^{\infty} \frac{1}{4+x^2}\ dx$
   b. $\int_0^2 \frac{1}{(x-1)^2}\ dx$
   c. For what value(s) of p is $\int_1^{\infty} \frac{1}{x^{2p}}\ dx$ convergent? Justify your answer.
4. Evaluate the limits. [6]
   a. $\lim_{\theta\to 0} \frac{2\theta-\sin(2\theta)}{\theta-\sin(\theta)}$
   b. $\lim_{x\to 0^+} (1+\sin(2x))^{1/x}$
5. Find the area of the region bounded by $y = \frac{4}{x}$ and $y = 5 - x$ [4]
6. In the diagram below, there are two triangular regions. [7]
   - Let $\mathcal{R}$ be the triangular region in Quadrant I (region bounded by the graph $y = x$ and $y = \frac{x}{3}$, between $x = 0$ and $x = 2$)

- Let S be the triangular region in quadrant III (region bounded by the graph $y = \frac{x}{3}$, $y = x$ and $y = -1$)

In each part below express (do not evaluate) using one integral , the volume of the solid of revolution obtained by rotating

a. The region $\mathcal{R}$ about the y axis
b. The region S about the line $x = 3$



7. Find the length of the curve $y = 2x^{3/2} + 1$ from $x = 0$ to $x = \frac{1}{3}$ [4]

8. A tank contains 50 kg of salt dissolved in 1500L of water. Pure water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. [4]
   a. How much salt is in the tank after t minutes
   b. How much salt is in the tank after 150 minutes?

9. Given $a_n = \dfrac{3n^2 + \sin(n)}{5n^2 + n}$ [3]
   a. Does the sequence converge? Justify your answer
   b. Does $\sum_{n=1}^{\infty} a_n$ converge?

10. Determine whether each of the following series converges or diverges. Justify your answer. [6]
    a. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + 1}}$
    b. $\displaystyle\sum_{n=1}^{\infty} \left(1 - \cos\left(\frac{\pi}{2n}\right)\right)^n$

11. Determine whether each of the following series is absolutely convergent, conditionally convergent or divergent. Justify your answer. [6]
    a. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n+1)}{n+1}$
    b. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{n!}$

12. Determine whether each of the following series converges or diverges. If it converges find the sum [5]

a. $\sum_{n=0}^{\infty} \frac{2^{n+1}+7^n}{3^n}$

b. $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)}$

13. Find a formula for the n^{th} term of the Taylor series for $f(x) = \ln(1+x)$ centered at 1 [5]

14. Determine the radius and the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{n(x-2)^n}{n^2+1}$ [5]

Answers:

1) a) $\frac{1}{\sqrt{x^2+1}\sqrt{\left(\sqrt{x^2+1}\right)^2-1}} \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x$; b) $\frac{1}{x^2+1}$.

2) a) $\frac{3}{7}(x-1)^{7/3} + \frac{3}{4}(x-1)^{4/3} + c$; b) $2\left(\sqrt{3} - \frac{\pi}{3}\right)$; c) $\frac{t^3}{3}\arcsin t + \frac{\sqrt{1-t^2}}{9}\left(t^2+2\right) + c$

d) $\left(a^2-1\right)x + 2a\ln|\sec x| + \tan x + c$; e) $\arcsin(x-1) + c$; f) $2\ln|x| - \frac{5}{x} - \ln\left(x^2+1\right) - 4\arctan x + c$

3) a) $\frac{\pi}{2}$; b) The integral diverge; c) $p > \frac{1}{2}$

4) a) 8; b) e^2; 5) $\frac{15}{2} - 8\ln 2$ $units^2$; 6)a) $2\pi\int_0^2 x\left(x - \frac{x}{3}\right)dx$; b) $\pi\int_{-1}^{0} \left[(3-3y)^2 - (3-y)^2\right]dy$

7) $\frac{14}{27}$; 8)a) $y = 50e^{-t/150}$; b) $y = \frac{50}{e}$ kg; 9)a) $\frac{3}{5}$; b) The integral diverge by divergence test

10) a) Series Diverge by the Limit comparison test; b) The series converge by the Root test

11) a) The series is conditionally convergent (converge by A.S.T. , $|a_n|$ diverge by limit comparison test)

b) Absolutely convergent by Ratio test; 12) Geometric series with $r = \frac{7}{3} > 1$, so it diverge

b) telescoping sum $S_n = \frac{3}{4} - \frac{1}{n+1}$ which converge with a sum = $\frac{3}{4}$; 13) $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1}\frac{(x-1)^n}{2^n\, n}$

14) Radius of convergence = 1, interval of convergence $1 \le x < 3$.