(28) 1. Evaluate each of the following integrals without the use of integration tables.

(a) $\displaystyle\int \frac{5x^3\sin(x)+\sqrt{x}-10}{x^3}\,dx$ (b) $\displaystyle\int \frac{7x}{\sqrt{x+2}}\,dx$ (c) $\displaystyle\int (3x+2)^2\cos(5x)\,dx$

(d) $\displaystyle\int_0^8 \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}}\,dx$ (e) $\displaystyle\int \frac{x^6-9x^4+7x-6}{x^2-3x}\,dx$ (f) $\displaystyle\int_0^{\pi/4} \frac{\sec^2(x)}{\tan(x)+1}\,dx$

(g) $\displaystyle\int \frac{9x+7}{(x-2)(x+3)^2}\,dx$ (h) $\displaystyle\int_{-2}^{1} |x+1|\,dx$

(4) 2. Sketch the region bounded by the graphs of $f(x)=x^2+2x+1$ and $g(x)=2x+5$ and find its area.

(6) 3. A fast-food outlet finds that the demand function for its new side-dish is given by $p=\frac{32}{(x+1)^2}$ where p is the price in dollars per serving and x is the number of servings that can be sold per hour at this price. At the same time the franchise is prepared to sell x servings per hour at the price of p dollars and its supply function is given by $p=\frac{1}{2}(x+1)$. Find the equilibrium point and evaluate the consumer and producer surpluses.

(8) 4. Use the integration table to find the following indefinite integral. In each case, state the formula number and justify its use.

(a) $\displaystyle\int x\sqrt{x^4-9}\,dx$ (b) $\displaystyle\int \frac{1}{(x-1)\sqrt{x^2-2x+2}}\,dx$

(4) 5. Use Simpson's rule with n = 6 to approximate $\displaystyle\int_2^8 \sqrt{2^x-1}\,dx$. Round your answer to four decimal places.

(3) 6. Determine if the function $y=\ln(x^2+1)$ is a solution of the differential equation $y''e^y+xy'=\dfrac{2}{x^2+1}$.

7. Find the particular solution of the following differential equation with the given initial condition.

(4) (a) $\dfrac{1}{e^x}\dfrac{dy}{dx}=\dfrac{x}{y}$ with condition $y=5$ when $x=0$.

(4) (b) $y'=k(y-50)$ with conditions $y=60$ when $x=0$ and $y=130$ when $x=3$.

8. An investment fund is growing at a rate which is directly proportional to the time t (number of years after 2010) and inversely proportional to the amount of money invested in the fund. On January 1, 2010 there was \$100 in the fund and on January 1, 2012 there was \$150 in the fund.

(1) (a) Determine the differential equation.

(3) (b) Find the particular solution of the differential equation.

(1) (c) When will there be \$2350 in the fund?

(6) 9. Use l'Hôpital's rule to find each limit.

(a) $\displaystyle\lim_{x\to\pi}\frac{\cos\left(\frac{x}{2}\right)}{x-\pi}$ (b) $\displaystyle\lim_{x\to-\infty}\frac{2x^3+7x}{x^2-x^3}$

(6) 10. Determine whether or not each of the following improper integrals converges. Evaluate the integral if it converges.

(a) $\displaystyle\int_1^\infty \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ (b) $\displaystyle\int_0^2 \frac{x}{\sqrt{4-x^2}}\,dx$

(6) 11. Determine if the following sequence converges or diverges. If the sequence converges, find its limit.

(a) $a_n = \dfrac{5^n}{4^n}$ (b) $a_n = \dfrac{(-1)^{n+1}}{2^n}$

(3) 12. Write the general term a_n of the sequence $\left\{\frac{2}{3}, \frac{-4}{9}, \frac{8}{27}, \frac{-16}{81}, ...\right\}$

(6) 13. Determine with justification if the series converges or diverges. Find the sum if the series converges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n+3}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{5^{n+1}}$

(3) 14. Use a geometric series to express $1.\overline{24}$ as a quotient of two integers.

(4) 15. Mr. Gordon plans to invest \$300 at the end of each month for 15 years. If the account pays 9%, compounded monthly, how much will he have at the end of the 15 years?

Answers

1. (a) $-5\cos(x) - \frac{2}{3x^{\frac{3}{2}}} + \frac{5}{x^2} + C$

(b) $\frac{14}{3}(x+2)^{\frac{3}{2}} - 28\sqrt{x+2} + C$ or $14x\sqrt{x+2} - \frac{28}{3}(x+2)^{\frac{3}{2}} + C$ or $\frac{14}{3}\sqrt{x+2}(x-4) + C$

(c) $\frac{1}{5}(3x+2)^2\sin(5x) + \frac{6}{25}(3x+2)\cos(5x) - \frac{18}{125}\sin(5x) + C$ (d) $2e(e^2-1)$

(e) $\frac{1}{5}x^5 + \frac{3}{4}x^4 + 5\ln|x-3| + 2\ln|x| + C$

(f) $\ln 2$ (g) $\ln|x-2| - \ln|x+3| - \frac{4}{x+3} + C$ (h) $\frac{5}{2}$

2. $\frac{32}{3}$ **3.** Equilibrium point: $(3,2)$ Consumer surplus: 18 Producer surplus: $\frac{9}{4}$

4. (a) Formula 22, $\frac{1}{4}\left(x^2\sqrt{x^4-9} - 9\ln\left|x^2+\sqrt{x^4+9}\right|\right) + C$ (b) Formula 27, $\ln\left|\frac{\sqrt{x^2-2x+2}-1}{x-1}\right| + C$

5. 39.7510 **6.** Yes **7.** (a) $y^2 = 2xe^x - 2e^x + 27$ (b) $y = 50 + 10(8)^{\frac{x}{3}}$

8. (a) $\frac{dA}{dt} = k\frac{t}{A}$ (b) $A = \sqrt{3125t^2 + 10000}$ (c) On January 1, 2052 **9.** (a) $-\frac{1}{2}$ (b) -2

10. (a) Diverges (b) Converges to 2 **11.** (a) Diverges (b) Converges to 0

12. $a_n = (-1)^{n+1}\left(\frac{2}{3}\right)^n$ **13.** (a) Diverges (b) Converges to 1 **14.** $\frac{41}{33}$ **15.** \$114,373.14