(Marks)

(4) 1. Given the curves $f(x) = x^2 - 7$ and $g(x) = 1 - x^2$ ,

(a) determine the point(s) of intersection of $f(x)$ and $g(x)$,

(b) find the area of the region bounded by $f(x)$ and $g(x)$ from $x = -2$ to $x = 3$.

(6) 2. Given the demand function $p = 1000 - x^2$ and the supply function $p = 20x + 200$ ,

(a) find the equilibrium point,

(b) sketch and identify the regions representing the consumer and producer surpluses,

(c) evaluate the consumer's surplus.

3. Evaluate each of the following integrals without the use of integration tables.

(3) (a) $\displaystyle\int \frac{5x^5 - 4x^2 - 3x + 7}{2x^2}\,dx$

(4) (b) $\displaystyle\int \frac{x^3 - 10x - 15}{x^2 - 9}\,dx$

(3) (c) $\displaystyle\int (2x+1)\ln x\,dx$

(3) (d) $\displaystyle\int 3x\,e^{6x^2+7}\,dx$

(4) (e) $\displaystyle\int (3 - 2x^2)\sin(x)\,dx$

(4) (f) $\displaystyle\int_0^{\pi/2} 3\sin x\sqrt{\cos x}\,dx$

(4) (g) $\displaystyle\int_0^{1} x^2\sqrt{1-x}\,dx$

(4) (h) $\displaystyle\int \frac{x^2 + 10x - 23}{(x+1)(x-3)^2}\,dx$

(4) 4. Use Trapezoidal rule with $n = 5$ to approximate $\displaystyle\int_0^{2.5} \sqrt{1+x^4}\,dx$ .
Round your answer to three decimal places.

5. Use the table of integrals to solve each of the following.
In each case, state the formula number and justify its use.

(4) (a) $\displaystyle\int (x+7)^2\sqrt{x+10}\,dx$

(4) (b) $\displaystyle\int \frac{\sqrt{x^2+4x-5}}{(x+2)^2}\,dx$

(3) 6. Determine if the function $y = \sin(\ln x)$ is a solution of the differential equation $x^2y'' + xy' + y = 0$

(4) 7. Solve the differential equation $\dfrac{dy}{dx} = 3x^2\sqrt{y}$ for $y$ with the condition $y(-1) = 1$

(4) 8. The rate of spreading of fungus on trees in a forest is proportional to the square root of the number N of trees infected and is inversely proportional to time in months after the fungus is spotted. Suppose the number of trees found to be infected with fungus is 400 after one month ( t = 1 ) and 900 after two months. How many trees will be infected with fungus after 8 months?

(Marks)

9. Use l'Hôpital's rule to evaluate the following limits

(3) (a) $\lim\limits_{x \to 0} \dfrac{x\,\sin(x)}{e^x - e^{2x} + x}$

(3) (b) $\lim\limits_{x \to 1} \dfrac{\ln(2-x)}{1-x^2}$

10. Evaluate each improper integral and state whether it converges or diverges

(3) (a) $\displaystyle\int_1^{\infty} \frac{3}{(5+4x)^2}\,dx$

(3) (b) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{(\sin x)^3}\,dx$

(3) 11. Consider the sequence $\left\{\frac{3}{5}, \frac{-9}{9}, \frac{27}{13}, \frac{-81}{17}, \ldots\right\}$

(a) Find an expression for the $n^{th}$ term of the sequence.

(b) Give the $5^{th}$ term of the sequence.

12. Determine the convergence or divergence of each sequence $\{a_n\}$.
If the sequence converges, find the limit.

(3) (a) $a_n = \frac{2(n+3)!}{(n+1)!}$

(3) (b) $a_n = \frac{5^{n+1}}{7^n}$

(3) 13. Given the number $3.0\overline{12}$, express it using a geometric series, find the sum of the geometric series and write the number as the ratio of two integers.

14. Determine with *justification* if each of the following series is convergent or divergent.
If the series is convergent, *find its sum*.

(3) (a) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k}{5^{k-1}}$

(3) (b) $\displaystyle\sum_{k=1}^{\infty} \frac{3k-1}{\sqrt{k^2+7}}$

(4) 15. A deposit of \$100 is made at the beginning of every week for 15 years into a savings account that earns 5.2% a year compounded weekly. Find the balance in the account at the end of 15 years.

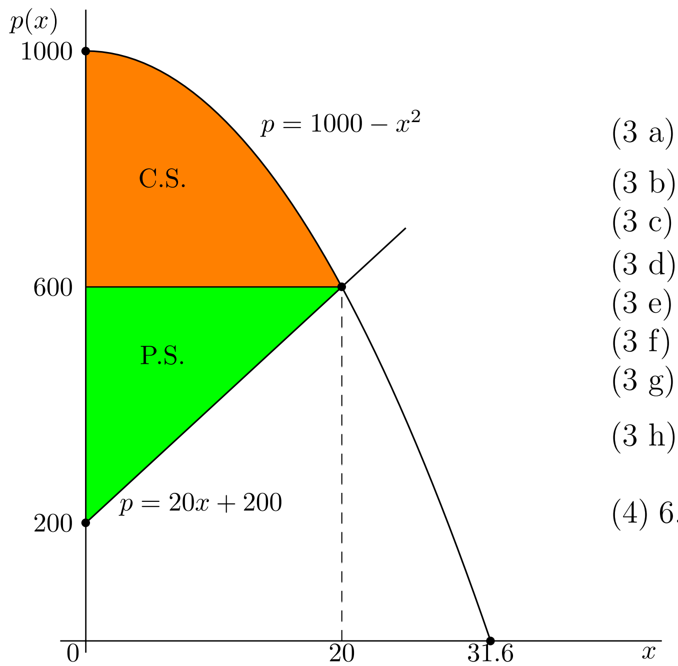
(4) 16. Given that the marginal cost function $\dfrac{dC}{dx} = \dfrac{300}{\sqrt{4x+5}}$ and the cost of producing 5 units is \$800.
Find the cost function.

(Marks)

# ANSWERS

(1 a) points of intersection: $(-2,-3)$ ; $(2,-3)$ ; (1 b) Area = 26 square units

(2 a) point of equilibrium: $(20, 600)$
(2 b)




(3 a) $\frac{5}{8}x^4 - 2x - \frac{3}{2}\ln|x| - \dfrac{7}{2x} + C$
(3 b) $\frac{1}{2}x^2 - 3\ln|x-3| + 2\ln|x+3| + C$
(3 c) $(x^2+x)\ln x - \frac{1}{2}x^2 - x + C$
(3 d) $\frac{1}{4}e^{6x^2+7} + C$
(3 e) $-(3-2x^2)\cos(x) - 4x\sin(x) - 4\cos(x) + C$
(3 f) 2
(3 g) $\frac{16}{105} \approx 0.15$
(3 h) $3\ln|x-3| - 2\ln|x+1| - \dfrac{4}{x-3} + C$

(4) 6.348

(2 c) C.S.=\$ 5333.33

(5 a) substitution then F14: $\frac{2}{105}\left(72 - 36(x+7) + 15(x+7)^2\right)(x+10)^{\frac{3}{2}} + C$

(5 b) complete the square then F25: $\frac{-\sqrt{x^2+4x-5}}{x+2} + \ln\left|x+2+\sqrt{x^2+4x-5}\right| + C$ ; (6) it is a solution

(7) $y = \left(\dfrac{x^3+3}{2}\right)^2 = \frac{1}{4}\left(x^3+3\right)^2$ ; (8) $N = 2500$

(9 a) $-\frac{2}{3}$ ; (9 b) $\frac{1}{2}$ ; (10 a) converges to $\frac{1}{12}$ ; (10 b) divergent

(11 a) $a_n = \frac{(-1)^{n+1}(3)^n}{4n+1}$ ; (11 b) $a_5 = \frac{81}{7}$

(12 a) divergent ; (12 b) converges to 0 ; (13) $\frac{497}{165}$

(14 a) converges to $\frac{15}{2}$ ; (14 b)divergent ; (15) \$118, 180.29 ; (16) $C = 150\sqrt{4x+5} + 50$