[6] 1.Solve the following linear system or show that it is inconsistent.

a) $\begin{cases} 2x - y + 3z = 1 \\ 4x + 2y + 5z = 4 \\ 2x + \quad 2z = 6 \end{cases}$ b) $\begin{cases} x + 2y - 2z \quad = -1 \\ x + y - z - w = 0 \\ x + y \quad - 5w = 3 \end{cases}$

[4] 2. Find the value(s) of a and b in the system $\begin{cases} x + 2y \quad = 3 \\ 4x + 7y + z = 10 \\ y - z = b \\ 2x + 3y + az = 4 \end{cases}$ such that the system has

a) a unique solution
b) infinitely many solutions
c) no solution.

[6] 3. A small company manufactures three different products. Product A needs 5 hours of labor, 9 kilograms of copper and 10 kilograms of steel. Product B needs 1 hour of labor, 2 kilograms of copper and 1 kilogram of steel. Product C needs 3 hours of labor, 5 kilograms of copper and 8 kilograms of steel. Each day, the company has available 13 hours of labor, 25 kilograms of copper and 18 kilograms of steel. How much of each product should be produced each day so that all of the resources are used?

a) Define all the necessary variables and set up the system of equations required to solve the problem.

b) Give all realistic solutions.

[7] 4. Let A = $\begin{bmatrix} 4 & 7 & 12 & 10 \\ 9 & -3 & 11 & 10 \end{bmatrix}$ , B = $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 4 & -1 & 4 & 3 \end{bmatrix}$, C = $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 4 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$. Find, if possible:

a) $BA^T$
b) $D^{-1} + 3I$
c) (A – 2B)C
d) AB

[4] 5. Let A = $\begin{bmatrix} 0 & 2 & 2 \\ 1 & -2 & -3 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

a) Find $A^{-1}$ by applying row operations.

b) Solve AX = B using $A^{-1}$.

[6] 6. Let a simple economy have two industries: Red and Green. The production of \$1 of Red requires 60¢ of Red and 20¢ of Green. The production of \$1 of Green requires 20¢ of Red and 40¢ of Green.

a) Find the production for the external demand D = $\begin{bmatrix} 1000 \\ 1100 \end{bmatrix}$.

b) Which, if either, of the two industries are profitable? Justify.

c) Is the economy productive? Justify.

d) What is the internal consumption?

[6] 7. Given A = $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, B = $\begin{bmatrix} a & b & c \\ \frac{d}{2} & \frac{e}{2} & \frac{f}{2} \\ 3g-2a & 3h-2b & 3i-2c \end{bmatrix}$, C = $\begin{bmatrix} 0 & 2 & -2 & 2 \\ 1 & 3 & -2 & 4 \\ 2 & 2 & -1 & 1 \\ -3 & 4 & 1 & 5 \end{bmatrix}$ and det(A) = 2 find

a) det(B)
b) $\det(AB)^{-1}$
c) $\det(3A^T I)$
d) det(C)

[8] 8. Given A = $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 0 & 2 \\ 4 & 3 & 1 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$ find

a) adj(A)
b) $A \cdot \text{adj}(A)$
c) det(A)
d) $A^{-1}$ using adj(A)
e) The value of y only for the system $A\vec{x} = \vec{b}$ using Cramer's Rule.

[3] 9. Given A = $\begin{bmatrix} 1 & 4 & 1 \\ 1 & 5 & k \\ 0 & 2 & 3 \end{bmatrix}$

a) find the value(s) of k, if any, such that the columns of A span $\Re^3$.

b) find the value(s) of k, if any, such that $A^{-1}$ does not exist.

[3] 10. If possible, express $\vec{u} = \begin{bmatrix} 2 \\ 4 \\ -7 \end{bmatrix}$ as a linear combination of $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 5 \\ 10 \\ -7 \end{bmatrix}$.

[5] 11. Given points P(-1, 2) and Q(3, 5)

a) find the vector $\overrightarrow{PQ}$

b) find the magnitude $\left\| 2\overrightarrow{PQ} \right\|$

c) find a vector equation for the line passing through point P and parallel to line L: 3x + 5y = 2.

d) Is the point (1, -1) on the line found in part c)?

[5] 12. Let S = {(x, y, z) $\varepsilon\, \Re^3$ |2x + y = 3}

a) Give two vectors in S.

b) Is $\vec{0}$ in S? Justify your answer.

c) Is S closed under vector addition? Justify your answer.

d) Is S closed under scalar multiplication? Justify your answer.

e) Is S a subspace of $\Re^3$?

[4] 13. Given $\vec{u} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -5 \\ 4 \\ 3 \end{bmatrix}$ and A = $\begin{bmatrix} 3 & -5 \\ -2 & 4 \\ 4 & 3 \end{bmatrix}$

a) Find a relation on x, y, z for a vector $\vec{b}$ = (x, y, z) to be in the span of $\vec{u}$ and $\vec{v}$.

b) Is Col(A) a line, plane, or $\Re^3$? Justify your answer.

[4] 14. Suppose A is a 4x9 matrix.

a) Is it possible for the equation $A\vec{x} = \vec{0}$ to have a unique solution? Explain.

b) Is it possible for the equation $A^T\vec{y} = \vec{0}$ to have a unique solution? Explain.

c) If there are 8 parameters in the solution $A\vec{x} = \vec{0}$, how many parameters are there in the solution to $A^T\vec{y} = \vec{0}$?

[8] 15. Let $A = \begin{bmatrix} 2 & 1 & 1 & 2 & 0 \\ 6 & 3 & 1 & 4 & 0 \\ 4 & 2 & 1 & 3 & 0 \\ -2 & -1 & 1 & 1 & 0 \end{bmatrix}$ and let $\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}, \overrightarrow{a_4}$ and $\overrightarrow{a_5}$ be the columns of A, in order from left to right.

a) Find a basis for Col(A).

b) For each of the following sets, state whether it is linearly dependent (LD) or linearly independent (LI). If it is LD, give a dependency equation.

i) $\{\overrightarrow{a_1}, \overrightarrow{a_2}, \overrightarrow{a_3}\}$

ii) $\{\overrightarrow{a_2}, \overrightarrow{a_3}, \overrightarrow{a_4}\}$

iii) $\{\overrightarrow{a_4}, \overrightarrow{a_5}\}$

c) Find a basis for Nul(A).

d) Is the vector $\vec{v} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ in Nul(A)? Justify your answer.

[7] 16. Solve the linear program by the simplex method. Give the full basic feasible solution for the final simplex table.

Maximize $z = 4x_1 + 9x_2 + 8x_3$

Subject to
$4x_1 + 3x_2 + x_3 \le 4$
$6x_1 + x_2 + 2x_3 \le 2$
$5x_1 - x_2 + 3x_3 \le 7$
$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

[7] 17. Use the simplex algorithm to show that the following linear program has no minimum. Give a full feasible solution with z = -5016.

Minimize $z = 4x_1 + 3x_2 - 2x_3$

Subject to
$-10x_1 - 2x_2 + x_3 \le 3$
$-4x_1 - x_2 + x_3 \le 2$
$10x_1 + x_2 - 2x_3 \le 2$
$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

[7] 18. J&J Jewelry has two Jewelers, Jack and Jill, who each earn $100 per day. In one day, Jack can make 2 rings, 1 bracelet, and 1 necklace. In one day, Jill can make 1 ring, 2 bracelets, and 4 necklaces. There is an order for 50 rings, 70 bracelets, and 80 necklaces. Determine the number of days Jack and Jill each have to work to minimize the total labor cost of the order.

Identify the variables, set-up a linear program that represents this situation, and solve by sketching the feasibility region.