1. (a) $k = 2$

   (b) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 5 \\ -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

2. LD, $-5\mathbf{u} - 2\mathbf{v} + \mathbf{w} = \mathbf{0}$

3. $2C_4H_{10} + 13H_2 \rightarrow 8CO_2 + 10H_2O$

4. (a) $2k - 9$

   (b) $k = 4$ or $k = 5$

5. (a) -100

   (b) 10

6. Using $\mathbf{u} = (0, 1)$ and $\mathbf{v} = (0, -1)$, we get $T(\mathbf{u} + \mathbf{v}) = (0, 0)$ and $T(\mathbf{u}) + T(\mathbf{v}) = (0, -2)$.

7. (a) Yes, because its standard matrix has a pivot in each column.

   (b) $\mathbf{x} = (36, -22)$

   (c) $\mathbb{R}^2$, because its standard matrix has a pivot in each row.

   (d) $\mathbf{v} = (18, -11, 1)$

   (e) $\begin{bmatrix} 11 & 19 & 11 \\ 18 & 31 & 17 \end{bmatrix}$

8. $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix}$

9. (a) $\det(ABC) = 1 \Rightarrow \det(A)\det(B)\det(C) = 1 \Rightarrow \det(A) \neq 0, \det(B) \neq 0, \det(C) \neq 0.$

   (b) $ABC = I \Rightarrow B = A^{-1}C^{-1} \Rightarrow B^{-1} = CA.$

10. $X^T = \left(D^TD + I\right)^T = \left(D^TD\right)^T + I^T = D^T\left(D^T\right)^T + I = D^TD + I = X.$

11. $\begin{bmatrix} 0 & 0 & -1 \\ 0 & \frac{1}{2} & 0 \\ 1 & -1 & 0 \end{bmatrix}$

12. (a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

    (b) -6

13. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 1 & 2 \\ 0 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

14. (a) $X = -BA^{-1}$, $Y = I$, $Z = A^{-1}$

    (b) $\begin{bmatrix} 7 & -18 & 1 & 0 \\ -21 & 54 & 0 & 1 \\ -3 & 8 & 0 & 0 \\ 2 & -5 & 0 & 0 \end{bmatrix}$

15. (a) i. $O\mathbf{b} = \mathbf{0} \Rightarrow O \in H.$

    ii. $A, B \in H \Rightarrow A\mathbf{b} = \mathbf{0}, B\mathbf{b} = \mathbf{0} \Rightarrow A\mathbf{b} + B\mathbf{b} = \mathbf{0} \Rightarrow (A + B)\mathbf{b} = \mathbf{0} \Rightarrow A + B \in H.$

    iii. $A \in H, k \in \mathbb{R} \Rightarrow A\mathbf{b} = \mathbf{0} \Rightarrow k(A\mathbf{b}) = k(\mathbf{0}) \Rightarrow (kA)\mathbf{b} = \mathbf{0} \Rightarrow kA \in H.$

(b) $\left\{ \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -2 & 1 \end{bmatrix} \right\}$

16. (a) It fails, because we have $\mathbf{u} = (1,0,0), \mathbf{v} = (0,1,0) \in S$, but $\mathbf{u}+\mathbf{v} = (1,1,0) \notin S$.

    (b) It is closed.
    $\mathbf{u} \in S, k \in \mathbb{R}$
    $\Rightarrow \mathbf{u} = (a,b,c)$ with $ab = c^2$
    $\Rightarrow k\mathbf{u} = (ka,kb,kc)$ and $k^2(ab) = k^2(c^2)$
    $\Rightarrow k\mathbf{u} = (ka,kb,kc)$ and $(ka)(kb) = (kc)^2$
    $\Rightarrow k\mathbf{u} \in S$

17. (a) $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

    (b) $\mathbf{v}_1 = \begin{bmatrix} -1 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

    (c) $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

    (d) $\dim(\operatorname{Col} A) = \dim(\operatorname{Row} B)$ and $\mathbf{w}_1, \mathbf{w}_2 \in \operatorname{Col} A$
    $\Rightarrow \{\mathbf{w}_1, \mathbf{w}_2\}$ is a basis for $\operatorname{Col} A$
    $\Rightarrow \operatorname{Col} A = \operatorname{span}\{\mathbf{w}_1, \mathbf{w}_2\} = \operatorname{Row} B$

18. (a) $\operatorname{Nul} A = \operatorname{Col} A$ and $\dim(\operatorname{Col} A) + \dim(\operatorname{Nul} A) = n$
    $\Rightarrow 2\dim(\operatorname{Col} A) = n$
    $\Rightarrow$ n is even

    (b) $A^2 = AA = [A\mathbf{a}_1 ... A\mathbf{a}_n] = [\mathbf{0}...\mathbf{0}] = 0$

19. (a) $\sqrt{147}$

    (b) $-11x_1 - 5x_2 + x_3 = 0$

    (c) 40 units cubed

    (d) $(-3, 0, 7)$

    (e) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

20. (a) $\frac{\sqrt{2}}{2}$

    (b) $(\frac{5}{3}, \frac{7}{3}, \frac{2}{3})$

21. (a) $\frac{1}{\sqrt{a^2+1}} \begin{bmatrix} a \\ -1 \end{bmatrix}$

    (b) $\frac{2a}{a^2+1} \begin{bmatrix} a \\ 1 \end{bmatrix}$

    (c) $\frac{2|a|}{a^2+1} \sqrt{a^2+1}$