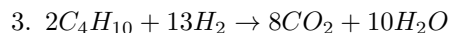


1. (a) $k = 2$

(b)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 5 \\ -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

2. LD, $-5\mathbf{u} - 2\mathbf{v} + \mathbf{w} = \mathbf{0}$



4. (a) $2k - 9$

(b) $k = 4$ or $k = 5$

5. (a) -100

(b) 10

6. Using $\mathbf{u} = (0, 1)$ and $\mathbf{v} = (0, -1)$, we get $T(\mathbf{u} + \mathbf{v}) = (0, 0)$ and $T(\mathbf{u}) + T(\mathbf{v}) = (0, -2)$.

7. (a) Yes, because its standard matrix has a pivot in each column.

(b) $\mathbf{x} = (36, -22)$

(c) \mathbb{R}^2 , because its standard matrix has a pivot in each row.

(d) $\mathbf{v} = (18, -11, 1)$

(e) $\begin{bmatrix} 11 & 19 & 11 \\ 18 & 31 & 17 \end{bmatrix}$

8.
$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{-\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{bmatrix}$$

9. (a) $\det(ABC) = 1 \Rightarrow \det(A)\det(B)\det(C) = 1 \Rightarrow \det(A) \neq 0, \det(B) \neq 0, \det(C) \neq 0$.

(b) $ABC = I \Rightarrow B = A^{-1}C^{-1} \Rightarrow B^{-1} = CA$.

10. $X^T = (D^T D + I)^T = (D^T D)^T + I^T = D^T (D^T)^T + I = D^T D + I = X$.

11.
$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & \frac{1}{2} & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

12. (a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

(b) -6

13.
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 1 & 2 \\ 0 & 2 & 3 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

14. (a) $X = -BA^{-1}, Y = I, Z = A^{-1}$

(b)
$$\begin{bmatrix} 7 & -18 & 1 & 0 \\ -21 & 54 & 0 & 1 \\ -3 & 8 & 0 & 0 \\ 2 & -5 & 0 & 0 \end{bmatrix}$$

15. (a) i. $O\mathbf{b} = \mathbf{0} \Rightarrow O \in H$.

ii. $A, B \in H \Rightarrow A\mathbf{b} = \mathbf{0}, B\mathbf{b} = \mathbf{0} \Rightarrow A\mathbf{b} + B\mathbf{b} = \mathbf{0} \Rightarrow (A + B)\mathbf{b} = \mathbf{0} \Rightarrow A + B \in H$.

iii. $A \in H, k \in \mathbb{R} \Rightarrow A\mathbf{b} = \mathbf{0} \Rightarrow k(A\mathbf{b}) = k(\mathbf{0}) \Rightarrow (kA)\mathbf{b} = \mathbf{0} \Rightarrow kA \in H$.

$$(b) \left\{ \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -2 & 1 \end{bmatrix} \right\}$$

16. (a) It fails, because we have $\mathbf{u} = (1, 0, 0)$, $\mathbf{v} = (0, 1, 0) \in S$, but $\mathbf{u} + \mathbf{v} = (1, 1, 0) \notin S$.

(b) It is closed.

$$\mathbf{u} \in S, k \in \mathbb{R}$$

$$\Rightarrow \mathbf{u} = (a, b, c) \text{ with } ab = c^2$$

$$\Rightarrow k\mathbf{u} = (ka, kb, kc) \text{ and } k^2(ab) = k^2(c^2)$$

$$\Rightarrow k\mathbf{u} = (ka, kb, kc) \text{ and } (ka)(kb) = (kc)^2$$

$$\Rightarrow k\mathbf{u} \in S$$

$$17. (a) \mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$(b) \mathbf{v}_1 = \begin{bmatrix} -1 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$(c) \mathbf{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(d) $\dim(\text{Col } A) = \dim(\text{Row } B)$ and $\mathbf{w}_1, \mathbf{w}_2 \in \text{Col } A$

$$\Rightarrow \{\mathbf{w}_1, \mathbf{w}_2\} \text{ is a basis for } \text{Col } A$$

$$\Rightarrow \text{Col } A = \text{span}\{\mathbf{w}_1, \mathbf{w}_2\} = \text{Row } B$$

18. (a) $\text{Nul } A = \text{Col } A$ and $\dim(\text{Col } A) + \dim(\text{Nul } A) = n$

$$\Rightarrow 2 \dim(\text{Col } A) = n$$

$$\Rightarrow n \text{ is even}$$

$$(b) A^2 = AA = [A\mathbf{a}_1 \dots A\mathbf{a}_n] = [\mathbf{0} \dots \mathbf{0}] = \mathbf{0}$$

19. (a) $\sqrt{142}$

$$(b) -11x_1 - 5x_2 + x_3 = 0$$

(c) 40 units cubed

(d) $(-3, 0, 7)$

$$(e) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

20. (a) $\frac{\sqrt{2}}{2}$

$$(b) \left(\frac{5}{3}, \frac{7}{3}, \frac{2}{3}\right)$$

$$21. (a) \frac{1}{\sqrt{a^2+1}} \begin{bmatrix} a \\ -1 \end{bmatrix}$$

$$(b) \frac{2a}{a^2+1} \begin{bmatrix} a \\ 1 \end{bmatrix}$$

$$(c) \frac{2|a|}{a^2+1} \sqrt{a^2+1}$$