(3) 1. Given $f(x) = \arctan\left(\frac{1}{x+1}\right)$, find $f'(x)$ and simplify your answer.

2. Evaluate the following integrals.

(3) (a) $\int \frac{x^2}{\sqrt{x-4}} dx$

(4) (b) $\int \frac{x \arcsin(x^2)}{\sqrt{1-x^4}} dx$

(4) (c) $\int_0^{\pi/4} \sqrt{\tan x} \sec^4 x dx$

(5) (d) $\int (\cos^2\theta + \sin^3\theta) d\theta$

(4) (e) $\int \frac{\sqrt{9x^2-4}}{x} dx$

(4) (f) $\int \frac{6x^2-5x-1}{(x-2)(x^2+9)} dx$

(4) (g) $\int 16x(\arctan(4x)) dx$

(9) 3. Evaluate each of the following limits, using ∞ and $-\infty$ when appropriate. Justify!

(a) $\lim_{x\to 0^+} \tan x \ln x$

(b) $\lim_{x\to 3^-} \left[\frac{1}{\ln(x-2)} - \frac{1}{x-3}\right]$

(c) $\lim_{x\to\infty} (e^{2x} + e^{-2x})^{1/x}$

(8) 4. Evaluate each of the following improper integrals.

(a) $\int_6^{\infty} \frac{dx}{x\sqrt{x^2-9}}$

(b) $\int_0^{10} \frac{dx}{(x-4)^2}$

(5) 5. Let $\mathcal{R}_1$ be the region bounded by $y = \sin x$ and $y = \sqrt{3}\cos x$ between $x = 0$ and $x = \frac{\pi}{2}$.

(a) Sketch the region $\mathcal{R}_1$.

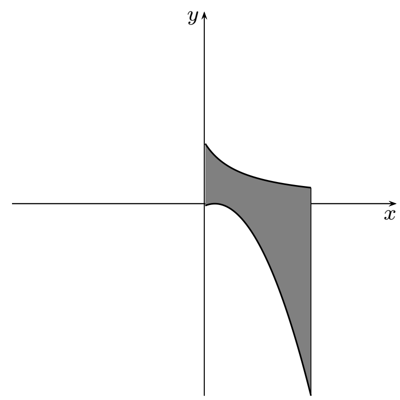
(b) Find the area of $\mathcal{R}_1$.

(3) 6. Solve the differential equation: $x - 8y\sqrt{x^2+1}\frac{dy}{dx} = 0; \quad y(0) = 1$

(5) 7. Let $\mathcal{R}_2$ be the region bounded by $y = \frac{1}{x+1}$ and $y = -x^2$ between $x = 0$ and $x = 2$.

(a) Find the volume of the solid of revolution obtained by rotating the region $\mathcal{R}_2$ about the y-axis.

(b) Set up but **do not evaluate** the integral for the volume obtained by rotating $\mathcal{R}_2$ about the line $y = 3$.



(2) 8. Does the sequence $\{\frac{n^3}{n!}\}_{n=1}^{\infty}$ converge? If so find its limit, if not explain why.

(4) 9. One particular increasing sequence $\{a_n\}_{n=1}^{\infty}$ is defined by the following properties:

$$a_1 = 1, \qquad a_{n+1} = 3 - \frac{1}{a_n}$$

(a) Find the first 4 terms of the sequence.

(b) Give an upper bound for the sequence.

(c) What theorem justifies that the sequence converges?

(d) Find $\lim_{n\to\infty} a_n$.

(6) 10. Determine whether the series converges or diverges. Find the sum if it converges.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^{n-1}}{3^{n+1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

11. Determine whether the series converges or diverges. State the tests that you use and justify your answers.

(2) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{-n^5 + n^2 + 1}{n^5 + 2}$

(3) (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(3) (c) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{4n^3 - 5}$

(3) (d) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

12. Determine whether the series converges absolutely, conditionally or diverges. Justify your answers.

(2) (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$

(2) (b) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2 + 5n}$

(3) (c) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{n^3}$

(4) 13. Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{e^n}{n^3}(x-4)^n$

(5) 14. Let $f(x) = \frac{1}{(1+x)^3}$.

(a) Write the first four nonzero terms of the Maclaurin series of f.

(b) Find a formula for the n^{th} nonzero term of the Maclaurin series, and write the series using sigma notation.